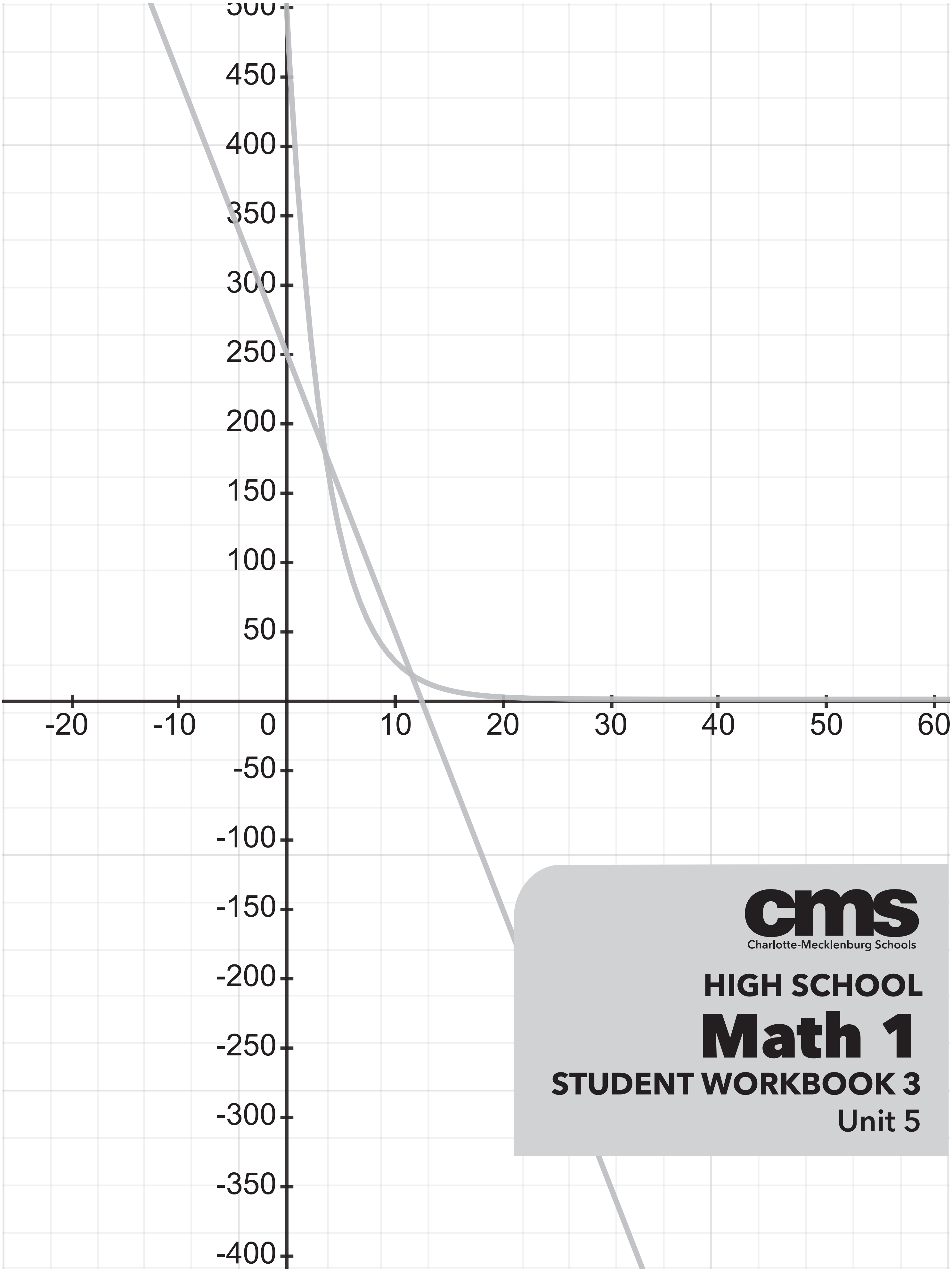


Charlotte-Mecklenburg Schools

HIGH SCHOOL

Math 1

STUDENT WORKBOOK 3

Unit 5

Table of Contents

Lesson 1: Describing and Graphing Situations

Learning Targets

- I can make sense of descriptions and graphs of functions and explain what they tell us about situations.
- I can explain when a relationship between two quantities is a function.
- I can identify independent and dependent variables in a function and use words and graphs to represent the function.

Bridge

Mai is helping her band collect money to fund a field trip. The band decided to sell boxes of chocolate bars. Each bar sells for $1.50, and each box contains 20 bars. Below is a partial table of monies collected for different numbers of boxes sold.[1]

Boxes sold b	**Monies collected** m
1	$30.00
2	
3	
4	$120.00
5	
8	
20	
	$1530.00

a. Complete the table above.

b. Write an equation for the amount of money, m, that will be collected if b boxes of chocolate bars are sold.

c. Which is the independent variable and which is the dependent variable?

[1] Adapted from https://tasks.illustrativemathematics.org/

Warm-up: Bagel Shop

A customer at a bagel shop is buying 13 bagels. The shopkeeper says, “That will be $16.25.”

Jada, Priya, and Han, who are in the shop, all think it is a mistake.

- Jada says to her friends, “Shouldn’t the total be $13.25?”
- Priya says, “I think it should be $13.00.”
- Han says, “No, I think it should be $11.25.”

Explain how the shopkeeper, Jada, Priya, and Han could all be right.

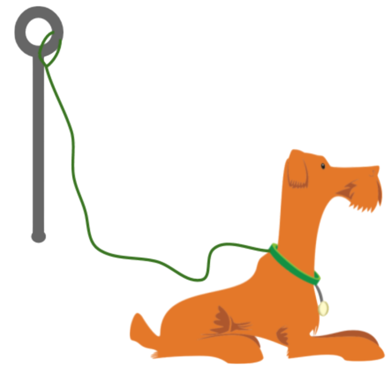
Work with a partner to complete the table:

Number of bagels	Best price
1	
2	
3	
4	
5	
6	
7	

Number of bagels	Best price
8	
9	
10	
11	
12	
13	

Activity 1: Be Right Back!

Three days in a row, a dog owner tied his dog's 5-foot-long leash to a post outside a store while he ran into the store to get a snack. Each time, the owner returned within minutes.



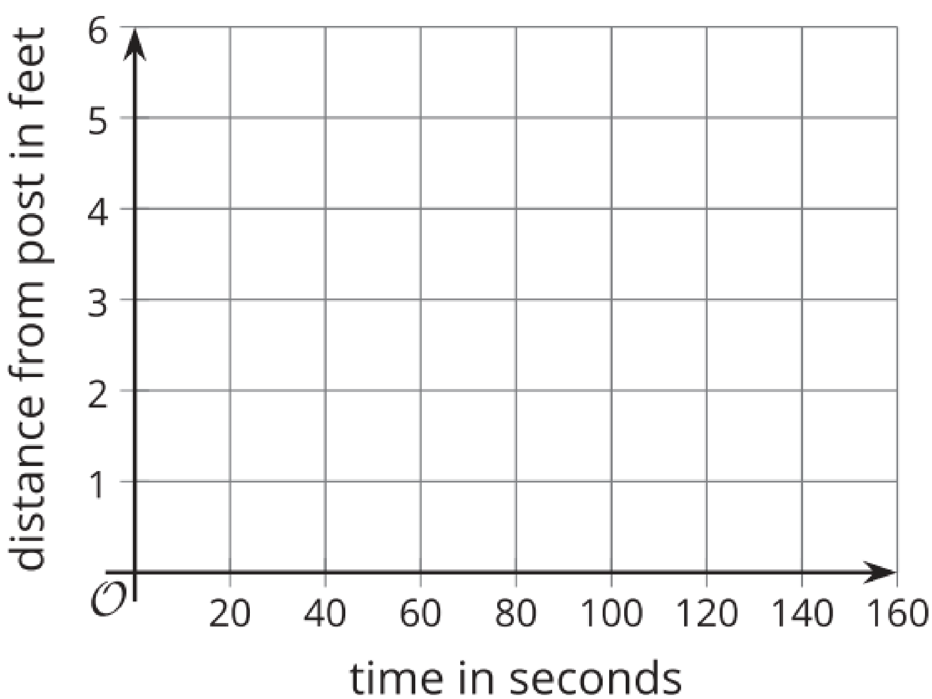
The dog's movement each day is described here.

- Day 1: The dog walked around the entire time while waiting for its owner.
- Day 2: The dog walked around for the first minute and then laid down until its owner returned.
- Day 3: The dog tried to follow its owner into the store but was stopped by the leash. Then, it started walking around the post in one direction. It kept walking until its leash was completely wound up around the post. The dog stayed there until its owner returned.
- Each day, the dog was 1.5 feet away from the post when the owner left.
- Each day, 60 seconds after the owner left, the dog was 4 feet from the post.

Your teacher will assign one of the days for you to analyze.

Sketch a graph that could represent the dog's distance from the post, in feet, as a function of time, in seconds, since the owner left.

Day ________



Are You Ready For More?

From the graph, is it possible to tell how many times the dog changed directions while walking around? Explain your reasoning.

Activity 2: Talk about a Function

Here are two pairs of quantities from a situation you've seen in this lesson. Each pair has a relationship that can be defined as a function.

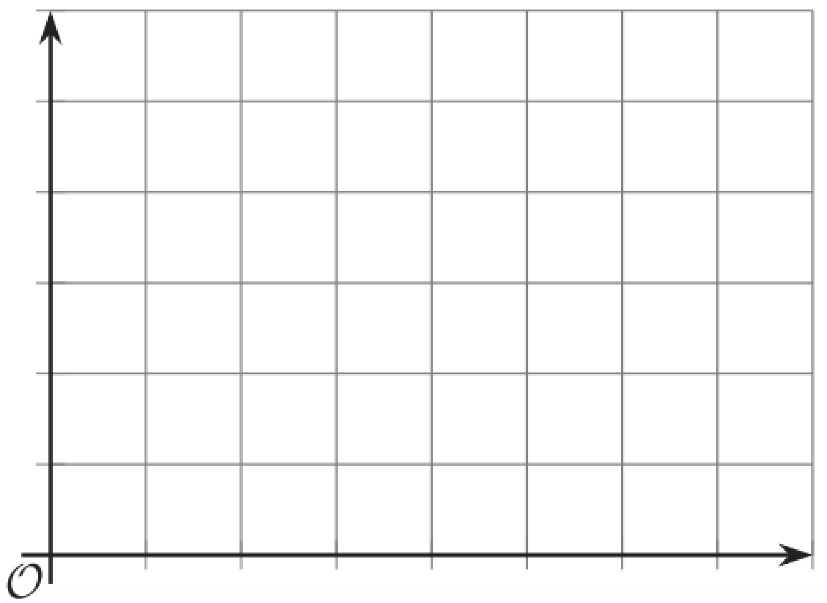
- Time, in seconds, since the dog owner left and the total number of times the dog has barked.
- Time, in seconds, since the owner left and the total distance, in feet, that the dog has walked while waiting.

Choose one pair of quantities and express their relationship as a function.

1. In that function, which variable is independent? Which one is dependent?

2. Write a sentence of the form "____ is a function of _____."

3. Sketch a possible graph of the relationship on the coordinate plane. Be sure to label and indicate a scale on each axis, and be prepared to explain your reasoning.



Lesson Debrief

Lesson 1 Summary and Glossary

A relationship between two quantities is a **function** if there is exactly one output for each input. We call the input the **independent variable** and the output the **dependent variable**.

Function: A function takes inputs from one set and assigns them to outputs from another set, assigning exactly one output to each input. For example, if apples cost $1.30 per pound, then the relationship between number of apples bought and price paid is a function. An input of 5 apples has an output of $6.50 and no other price.

Independent variable: A variable representing the input of a function. If apples cost $1.30 per pound, then the number of apples bought is the independent variable that determines the price paid for those apples.

Dependent variable: A variable representing the output of a function. If apples cost $1.30 per pound, then price paid for the apples is the dependent variable: it depends on the number of apples bought.

Let's look at the relationship between the amount of time since a plane takes off, in seconds, and the plane's height above the ground, in feet.

- These two quantities form a function if time is the independent variable (the input) and height is the dependent variable (the output). This is because at any amount of time since takeoff, the plane could only be at one height above the ground.

  For example, 50 seconds after takeoff, the plane might have a height of 180 feet. At that moment, it cannot be both 180 feet and 95 feet above the ground.

  For any input, there is only one possible output, so the height of the plane is *a function of* the time since takeoff.

- The two quantities do not form a function, however, if we consider height as the input and time as the output. This is because the plane can be at the same height for multiple lengths of times since takeoff.

  For instance, it is possible that the plane is 1,500 feet above ground when 300 seconds have passed AND when 425 seconds have passed. In fact, if the plane moves up and down a lot, there could be many other times at which the plane is 1,500 feet above ground.

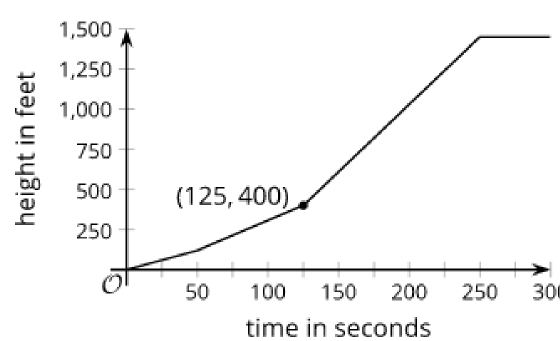
  For any input, there are multiple possible outputs, so the time since takeoff is *not a function of* the height of the plane.

Functions can be represented in many ways—with a verbal description, a table of values, a graph, an expression or an equation, or a set of ordered pairs.

When a function is represented with a graph, each point on the graph is a specific (input, output) pair..

Here is a graph that shows the height of a plane as a function of time since takeoff.

The point $(125, 400)$ tells us that 125 seconds after takeoff, the height of the plane is 400 feet.



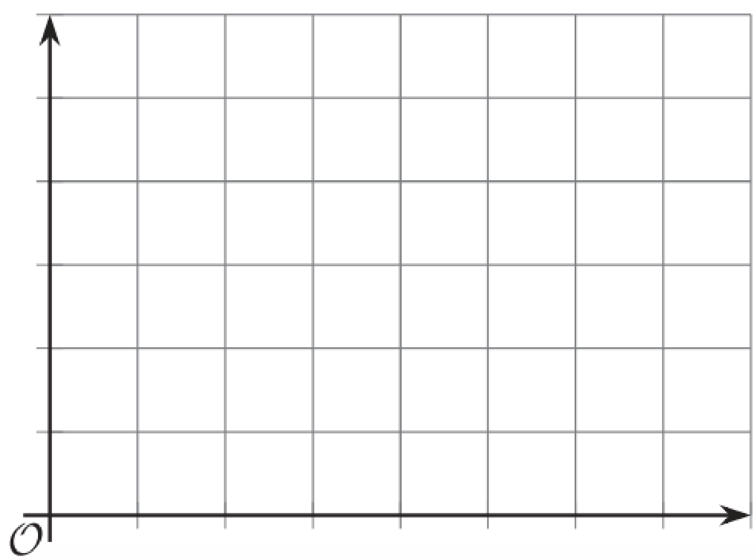
Unit 5 Lesson 1 Practice Problems

1. The relationship between the amount of time a car is parked, in hours, and the cost of parking, in dollars, can be described with a function.

   a. Identify the independent variable and the dependent variable in this function.

   b. Describe the function with a sentence of the form "_______________ is a function of ____________."

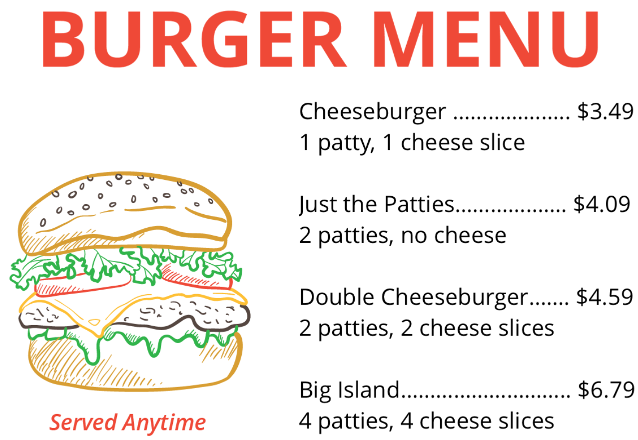
   c. Suppose it costs $3 per hour to park, with a maximum cost of $12. Sketch a possible graph of the function. Be sure to label the axes.



   d. Identify one point on the graph and explain its meaning in this situation.

2. The prices of different burgers are shown on this sign.

   Based on the information from the menu, is the price of a burger a function of the number of patties? Explain your reasoning.



3. The distance a person walks, d, in kilometers, is a function of time, t, in minutes, since the walk begins.

   Select **all** true statements about the input variable of this function.

   a. Distance is the input.
   b. Time of day is the input.
   c. Time since the person starts walking is the input.
   d. t represents the input.
   e. d represents the input.
   f. The input is not measured in any particular unit.
   g. The input is measured in hours.

4. It costs \$3 per hour to park in a parking lot, with a maximum cost of \$12.

   Explain why the amount of time a car is parked is not a function of the parking cost.

5. The Panthers, Charlotte's professional football team, track their score as a function of the amount of time passed in a football game. The table below gives the output scores, y, for a given time, x:

Time passed (in minutes)	0	10	20	30	40	50	60
Points scored	0	10	13	16	23	36	36

If the Panthers gain 7 points for each touchdown and 3 points for each field goal, write a brief story of their scoring in the game based on the function.

6. An airline company creates a scatter plot showing the relationship between the number of flights an airport offers and the average distance in miles travelers must drive to reach the airport. The correlation coefficient of the relationship between the two variables is -0.52.

   a. Describe the direction and strength of the correlation.

   b. Do either of the variables cause the other to change? Explain your reasoning.

(From Unit 4)

7. Here are clues for a puzzle involving two numbers.

    - Seven times the first number plus six times the second number equals 31.
    - Three times the first number minus ten times the second number is 29.

    What are the two numbers? Explain or show your reasoning.

    (From Unit 2)

8. Kiran shops for books during a 20% off sale.[2]

    a. What percent of the original price of a book does Kiran pay during the sale?

    b. Complete the table to show how much Kiran pays for books during the sale.

Original price in dollars, p	1	2	3	4	5	6
Sale price in dollars, s						

    c. Write an equation that relates the sale price, s, to the original price p.

    (Addressing NC.6.EE.9)

[2] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

Lesson 2: Function Notation

Learning Targets

- When given a statement written in function notation, I can explain what it means in terms of a situation.
- I understand what function notation is and why it exists.
- I can use function notation to express functions that have specific inputs and outputs.

Bridge

The following table shows the amount of garbage that was produced in the US each year between 2002 and 2010 as reported by the Environmental Protection Agency (EPA).[1]

t (years)	2002	2003	2004	2005	2006	2007	2008	2009	2010
G (million tons)	239	242	249	254	251	255	251	244	250

Let G be a function which assigns to an input t (a year between 2002 and 2010) the total amount of garbage produced in that year (in million tons). Use the table to answer each of the following questions.

1. How much garbage was produced in 2004?

2. In which year did the US produce 251 million tons of garbage?

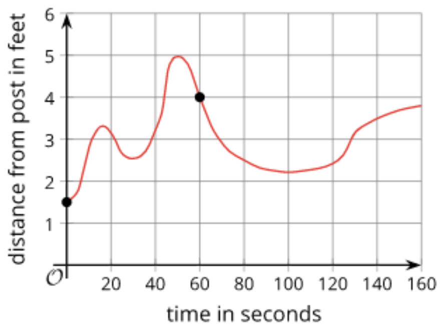
3. If the data in the table were graphed, what would the point $(2010, 250)$ represent in this context?

[1] Adapted from https://tasks.illustrativemathematics.org/

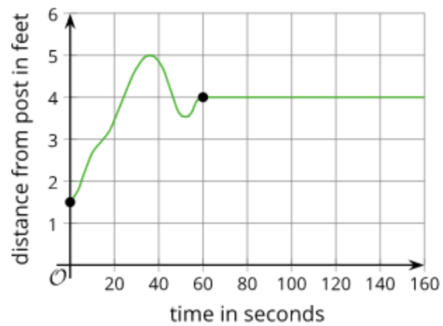
Warm-up: Back to the Post!

Here are the graphs of some situations you saw before. Each graph represents the distance of a dog from a post as a function of time since the dog owner left to purchase something from a store. Distance is measured in feet and time is measured in seconds.

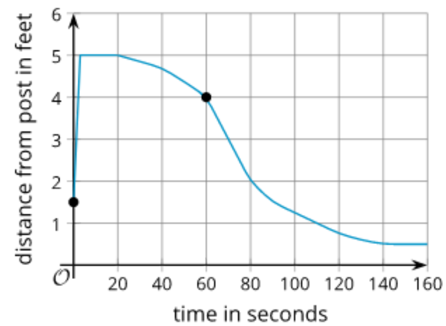
Day 1



Day 2



Day 3



1. Use the given graphs to answer these questions about each of the three days:

   a. How far away was the dog from the post 60 seconds after the owner left?

   Day 1: ______________ Day 2: ______________ Day 3: ______________

   b. How far away was the dog from the post when the owner left?

   Day 1: ______________ Day 2: ______________ Day 3: ______________

   c. The owner returned 160 seconds after he left. How far away was the dog from the post at that time?

   Day 1: ______________ Day 2: ______________ Day 3: ______________

   d. How many seconds passed before the dog reached the farthest point it could reach from the post?

   Day 1: ______________ Day 2: ______________ Day 3: ______________

2. Consider the statement, “The dog was 2 feet away from the post after 80 seconds.” Do you agree with the statement?

3. What was the distance of the dog from the post 100 seconds after the owner left?

Activity 1: A Handy Notation

Let's name the functions that relate the dog's distance from the post and the time since its owner left: function f for Day 1, function g for Day 2, function h for Day 3. The input of each function is time in seconds, t.

1. Use function notation to complete the table.

	Day 1	Day 2	Day 3
a. distance from post 60 seconds after the owner left	$f(60)$		
b. distance from post when the owner left			
c. distance from post 150 seconds after the owner left			

2. Describe what each expression represents in this context:

a. $f(15)$	b. $g(48)$	c. $h(t)$

3. The equation $g(120) = 4$ can be interpreted to mean: "On Day 2, 120 seconds after the dog owner left, the dog was 4 feet from the post." What does each equation mean in this situation?

a. $h(40) = 4.6$	b. $f(t) = 5$	c. $g(t) = d$

Activity 2: Birthdays

Rule B takes a person's name as its input and gives their birthday as the output.

Rule P takes a date as its input and gives a person with that birthday as the output.

Rule B

Input	Output
Abraham Lincoln	February 12

Rule P

Input	Output
August 26	Katherine Johnson

1. Complete each table with three more examples of input-output pairs.
2. If you use your name as the input to B, how many outputs are possible? Explain how you know.
3. If you use your birthday as the input to P, how many outputs are possible? Explain how you know.
4. Only one of the two relationships is a function. The other is not a function. Which one is which? Explain how you know.
5. For the relationship that is a function, write two input-output pairs from the table using function notation.

Are You Ready For More?

1. Write a rule that describes these input-output pairs:

   $F(ONE) = 3$ $F(TWO) = 3$ $F(THREE) = 5$ $F(FOUR) = 4$

2. Here are some input-output pairs with the same inputs but different outputs:

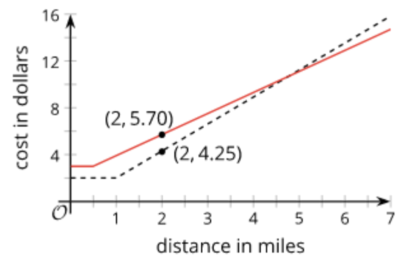
   $v(ONE) = 2$ $v(TWO) = 1$ $v(THREE) = 2$ $v(FOUR) = 2$

   What rule could define function v?

Lesson Debrief

Lesson 2 Summary and Glossary

Here are graphs of two functions, each representing the cost of riding in a taxi from two companies: Friendly Rides and Great Cabs.



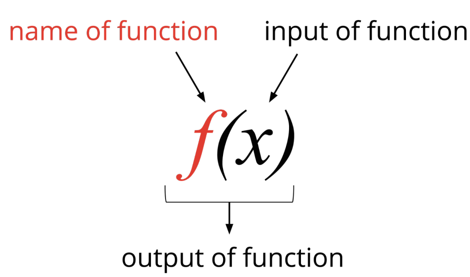
For each taxi, the cost of a ride is a function of the distance traveled. The input is distance in miles, and the output is cost in dollars.

- The point $(2, 5.70)$ on one graph tells us the cost of riding a Friendly Rides taxi for 2 miles.
- The point $(2, 4.25)$ on the other graph tells us the cost of riding a Great Cabs taxi for 2 miles.

We can convey the same information much more efficiently by naming each function and using **function notation** to specify the input and the output.

- Let's name the function for Friendly Rides function f.
- Let's name the function for Great Cabs function g.
- To refer to the cost of riding each taxi for 2 miles, we can write: $f(2)$ and $g(2)$.
- To say that a 2-mile trip with Friendly Rides will cost \$5.70, we can write $f(2) = 5.70$.
- To say that a 2-mile trip with Great Cabs will cost \$4.25, we can write $g(2) = 4.25$.

In general, function notation has this form:



It is read "f of x" and can be interpreted to mean: $f(x)$ is the output of a function f when x is the input.

The function notation is a concise way to refer to a function and describe its input and output, which can be very useful. Throughout this unit and the course, we will use function notation to talk about functions.

Function notation: A particular way of writing the outputs of a function that you have given a name to. If the function is named f and x is an input, then $f(x)$ denotes the corresponding output.

Unit 5 Lesson 2 Practice Problems

1. The height of water in a bathtub, w, is a function of time, t. Let P represent this function. Height is measured in inches and time in minutes. Match each statement in function notation with a description.

   a. $P(0) = 0$
   b. $P(4) = 10$
   c. $P(10) = 4$
   d. $P(20) = 0$

   1. After 20 minutes, the bathtub is empty.
   2. The bathtub starts out with no water.
   3. After 10 minutes, the height of the water is 4 inches.
   4. The height of the water is 10 inches after 4 minutes.

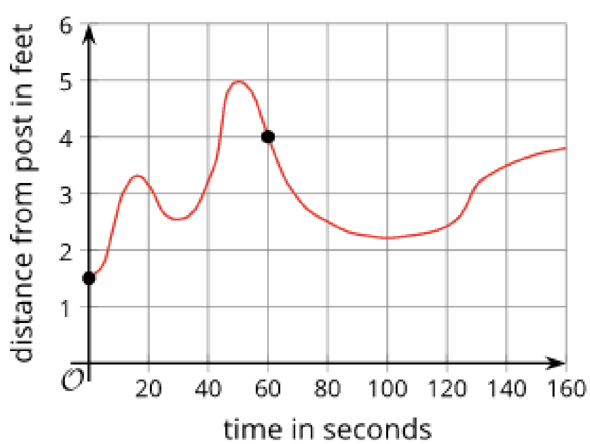
2. Function C takes time for its input and gives a student's Monday class for its output.

   a. Use function notation to represent: A student has English at 10:00.

   b. Write a statement to describe the meaning of $C(11{:}15) = \text{chemistry}$.

3. Function f gives the distance of a dog from a post, in feet, as a function of time, in seconds, since its owner left.

   Find the approximate values of $f(20)$ and of $f(140)$.



4. Function C gives the cost, in dollars, of buying n apples. What does each expression or equation represent in this situation?

   a. $C(5) = 4.50$

   b. $C(2)$

5. A number of identical cups are stacked up. The number of cups in a stack and the height of the stack in centimeters are related.

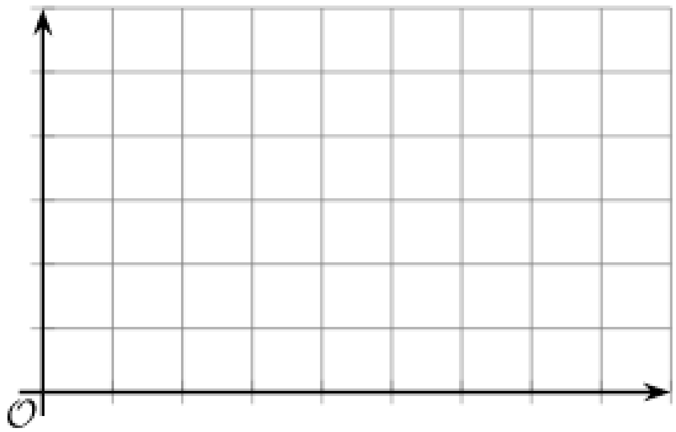
   a. Can we say that the height of the stack is a function of the number of cups in the stack? Explain your reasoning.

   b. Can we say that the number of cups in a stack is a function of the height of the stack? Explain your reasoning.

(From Unit 5, Lesson 1)

6. In a function, the number of cups in a stack is a function of the height of the stack in centimeters.

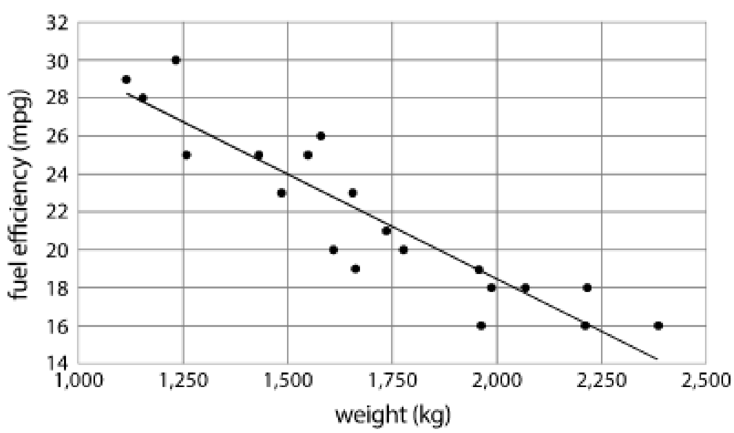
   a. Sketch a possible graph of the function on the coordinate plane. Be sure to label the axes.



   b. Identify one point on the graph and explain the meaning of the point in the situation.

(From Unit 5, Lesson 1)

7. The graph below describes various cars at a car dealership, and the line $y = -0.011x + 40.604$ is a line of best fit. Interpret the slope and y-intercept based on the labels of the x and y axes.[2]



(From Unit 4)

[2] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

8. Solve each system of equations without graphing. Show your reasoning.

a. $\begin{cases} -5x + 3y = -8 \\ 3x - 7y = -3 \end{cases}$

b. $\begin{cases} -8x - 2y = 24 \\ 5x - 3y = 2 \end{cases}$

(From Unit 3)

9. Jada and Kiran are solving the inequality $4x - 6 > 7x + 8$. Their steps are listed below:

Jada

$$\begin{aligned} 4x - 6 &> 7x + 8 \\ -3x - 6 &> 8 \\ -3x &> 14 \\ x &> -\tfrac{14}{3} \end{aligned}$$

Kiran

$$\begin{aligned} 4x - 6 &> 7x + 8 \\ -6 &> 3x + 8 \\ -14 &> 3x \\ -\tfrac{14}{3} &> x \end{aligned}$$

Do you agree with Jada, Kiran, both, or neither? Explain your reasoning.

(From Unit 2)

10. The table below represents the population of India by decade.

Year	1960	1970	1980	1990	2000	2010	2020
Population (in millions)	451	555	699	873	1,057	1,234	1,380

Let P be a function which assigns to an input t (a year between 1960 and 2020) the population in that year (in millions of people). Use the table to answer each of the following questions.

a. What was the population of India in the year 2000?

b. In which year was the population about 700 million people?

c. If the data in the table were graphed, what would the point $(2020, 1380)$ represent?

(Addressing NC.8.F.1)

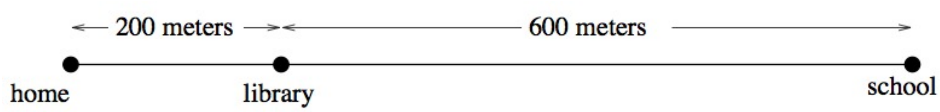
Lesson 3: Interpreting and Using Function Notation

Learning Targets

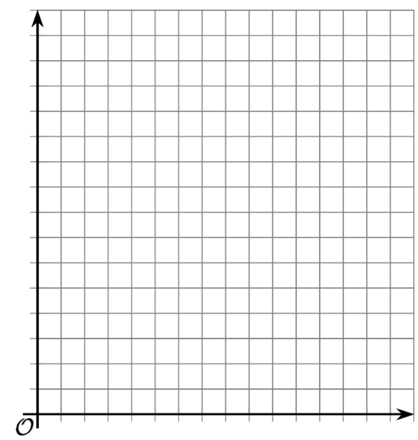
- I can describe the connections between a statement in function notation and the graph of the function.
- I can use function notation to efficiently represent a relationship between two quantities in a situation.
- I can use statements in function notation to sketch a graph of a function.

Bridge

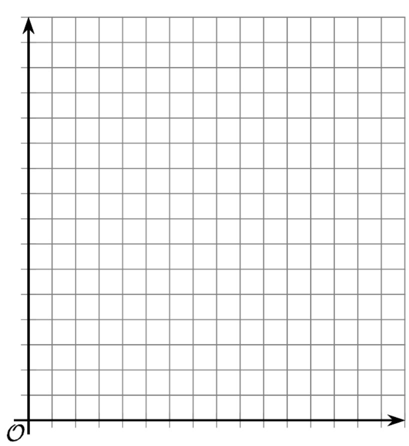
Nina rides her bike from her home to school, passing by the library on the way and traveling at a constant speed for the entire trip.[1] (See map below.)



1. Sketch a graph of Nina's distance from school as a function of time.



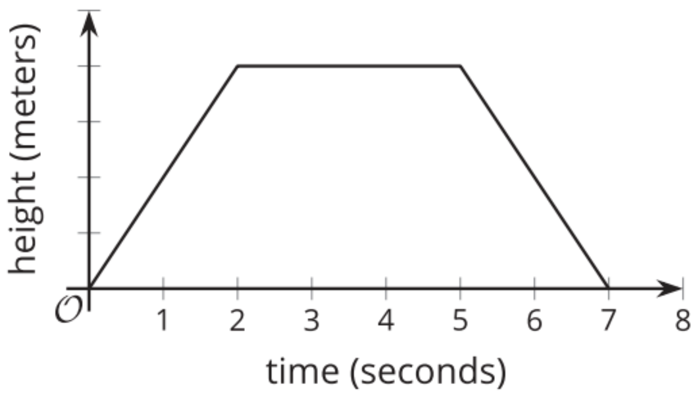
2. Sketch a graph of Nina's distance from the library as a function of time.



[1] Adapted from https://tasks.illustrativemathematics.org/

Warm-up: Observing a Drone

Here is a graph that represents function f, which gives the height of a drone, in meters, t seconds after it leaves the ground.



Decide which function value is greater, if they are equal, or if we can’t tell. Explain your reasoning.

1. $f(0)$ or $f(4)$
2. $f(2)$ or $f(5)$
3. $f(3)$ or $f(7)$
4. $f(t)$ or $f(t+1)$

Activity 1: Smartphones

The function P gives the number of people, in millions, who own a smartphone t years after the year 2000.

1. What does each equation tell us about smartphone ownership?
   a. $P(17) = 2,320$
   b. $P(-10) = 0$

2. Use function notation to represent each statement.

   a. In 2010, the number of people who owned a smartphone was 296,600,000.

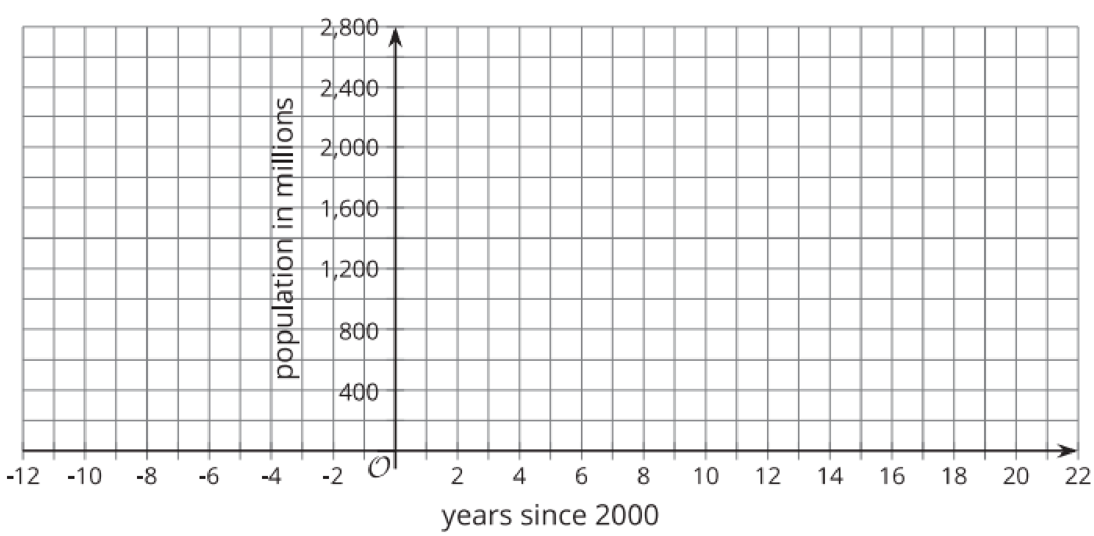
   b. In 2015, about 1.86 billion people owned a smartphone.

3. Mai is curious about the value of t in $P(t) = 1,000$.

   a. What would the value of t tell Mai about the situation?

   b. Is 4 a possible value of t here?

4. Use the information you have so far to sketch a graph of the function.



Are You Ready For More?

What can you say about the value or values of t when $P(t) = 1,000$?

Activity 2: Boiling Water

The function W gives the temperature, in degrees Fahrenheit, of a pot of water on a stove t minutes after the stove is turned on.

1. Take turns with your partner to explain the meaning of each statement in this situation. When it's your partner's turn, listen carefully to their interpretation. If you disagree, discuss your thinking and work to reach an agreement.

    a. $W(0) = 72$

    b. $W(5) > W(2)$

    c. $W(10) = 212$

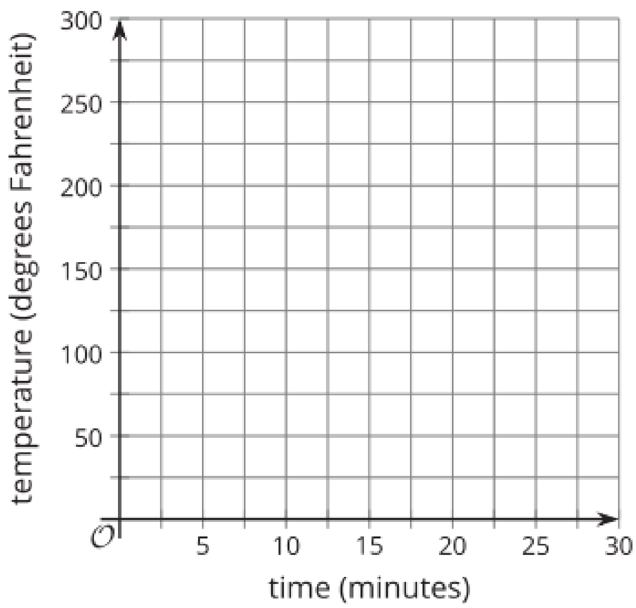
    d. $W(12) = W(10)$

    e. $W(15) > W(30)$

    f. $W(30) - W(20) = -75$

2. If all statements in the previous question represent the situation, sketch a possible graph of function W.

   Be prepared to show where each statement can be seen on your graph.



Are You Ready For More?

A family went on a hiking trip, and the temperature changed drastically as they traveled up the mountain. The function C gives the temperature, in degrees Celsius, of the air t minutes after the family started hiking. Explain the meaning of each statement in this situation.

a. $C(0) = 11$

b. $C(45) > C(90)$

c. $C(120) = -3$

d. When $100 < t < 130$, $C(t) < 0$. Otherwise, $C(t) \geq 0$.

e. Draw a graph that could represent this function.

Lesson Debrief

Lesson 3 Summary and Glossary

What does a statement like $p(3) = 12$ mean?

On its own, $p(3) = 12$ tells us that when p takes 3 as its input, its output is 12. The point $(3, 12)$ is on the graph of the function p.

If we know what quantities the input and output represent, we can learn more about the situation that the function represents.

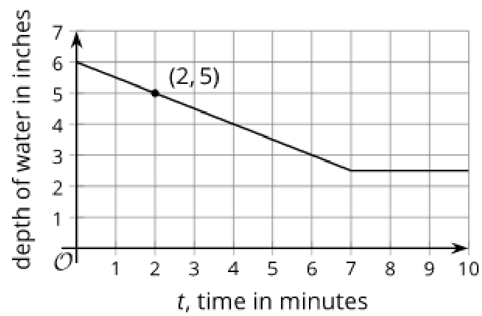
- If function p gives the perimeter of a square whose side length is x and both measurements are in inches, then we can interpret $p(3) = 12$ to mean "a square whose side length is 3 inches has a perimeter of 12 inches."
- We can also interpret statements like $p(x) = 32$ to mean "a square with side length x has a perimeter of 32 inches," which then allows us to reason that x must be 8 inches and to write $p(8) = 32$.
- If function p gives the number of blog subscribers, in thousands, x months after a blogger started publishing online, then $p(3) = 12$ means "3 months after a blogger started publishing online, the blog has 12,000 subscribers."

It is important to pay attention to the units of measurement when analyzing a function. Otherwise, we might mistake what is happening in the situation. If we miss that $p(x)$ is measured in thousands, we might misinterpret $p(x) = 36$ to mean "there are 36 blog subscribers after x months," while it actually means "there are 36,000 subscribers after x months."

A graph of a function can likewise help us interpret statements in function notation.

Function f gives the depth, in inches, of water in a tub as a function of time, t, in minutes, since the tub started being drained.

Here is a graph of f. Each point on the graph has the coordinates $(t, f(t))$, where the first value is the input of the function and the second value is the output.



- $f(2)$ represents the depth of water 2 minutes after the tub started being drained. The graph passes through $(2, 5)$, so the depth of water is 5 inches when $t = 2$. The equation $f(2) = 5$ captures this information.
- $f(0)$ gives the depth of the water when the draining began, when $t = 0$. The graph shows the depth of water to be 6 inches at that time, so we can write $f(0) = 6$.
- $f(t) = 3$ tells us that t minutes after the tub started draining, the depth of the water was 3 inches. The graph shows that this happens when t is 6.
- $f(7) = f(10)$ tells us that at 7 minutes and at 10 minutes after the tub started draining, the depth of the water was the same. The graph shows that at both times the depth is 2.5 inches. This can be written as $f(7) = 2.5$ and $f(10) = 2.5$.
- $f(4) - f(2) = -1$ tells us that the depth of the water 4 minutes after the tub started draining is one inch lower than the depth of the water 2 minutes after the tub started draining. The graph shows that $f(4) = 4$ and $f(2) = 5$ so $f(4) - f(2) = 4 - 5 = -1$.

Unit 5 Lesson 3 Practice Problems

1. Function f gives the temperature, in degrees Celsius, t hours after midnight.

   Choose the equation that represents the statement: "At 1:30 p.m., the temperature was 20 degrees Celsius."

   a. f(1:30)$= 20$

   b. $f(1.5) = 20$

   c. f(13:30)$= 20$

   d. $f(13.5) = 20$

2. Tyler filled up their bathtub, took a bath, and then drained the tub. The function B gives the depth of the water, in inches, t minutes after they began to fill the bathtub.

   Explain the meaning of each statement in this situation.

   a. $B(0) = 0$

   b. $B(1) < B(7)$

   c. $B(9) = 11$

   d. $B(10) = B(22)$

   e. $B(40) - B(20) = -13$

3. Function f gives the temperature, in degrees Celsius, t hours after midnight.

   Use function notation to write an equation or expression for each statement.

   a. The temperature at 12 p.m.

   b. The temperature was the same at 9 a.m. and at 4 p.m.

   c. It was warmer at 9 a.m. than at 6 a.m.

   d. Some time after midnight, the temperature was 24 degrees Celsius.

4. Select **all** points that are on the graph of f if we know that $f(2) = -4$ and $f(5) = 3.4$.

   a. $(-4, 2)$

   b. $(2, -4)$

   c. $(3.4, 5)$

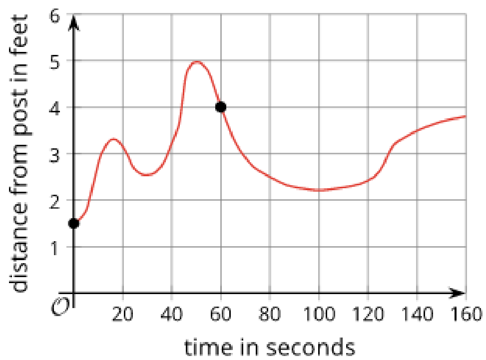
   d. $(5, 3.4)$

   e. $(2, 5)$

5. Write three statements that are true about this situation. Use function notation.

   Function f gives the distance of a dog from a post, in feet, as a function of time, t, in seconds, since its owner left.

   Use the $=$ sign in at least one statement and the $<$ sign in another statement.



6. The function $A(x)$, where x is a letter, assigns a value based on the number of the letter in the order of the alphabet. For example, $A(A) = 1$, $A(B) = 2$, $A(C) = 3$, etc. Diego and Andre are trying to figure out whose name as a higher value based on the function $A(x)$ by figuring out the value of each letter in their names and adding them. Whose name has a higher sum based on the $A(x)$ function, Diego or Andre?

(From Unit 5, Lesson 2)

7. A restaurant owner wants to see if there is a relationship between the amount of sugar in some food items on her menu and how popular the items are.

   She creates a scatter plot to show the relationship between amount of sugar in menu items and the number of orders for those items. The correlation coefficient for the relationship between the two variables is 0.58.

   a. Describe the relationship between the two variables.

   b. Does either of the variables cause the other to change? Explain your reasoning.

(From Unit 4)

8. Elena writes the equation $6x + 2y = 12$. Write a new equation that has:

   a. exactly one solution in common with Elena's equation

   b. no solutions in common with Elena's equation

   c. infinitely many solutions in common with Elena's equation

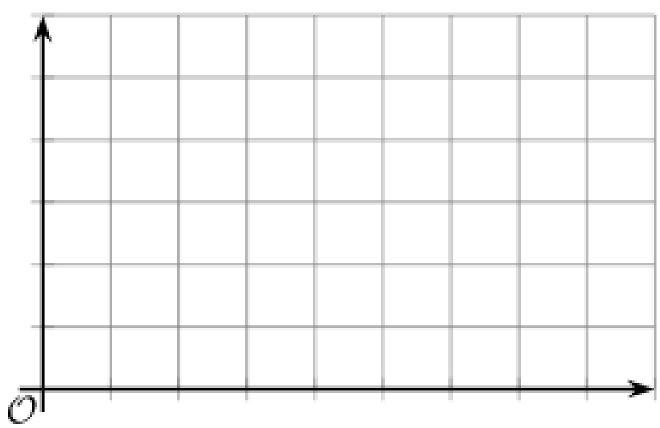
(From Unit 3)

9. Solve the equation: $\frac{2}{3}(12x - 15) + 8 = \frac{1}{4}(8x + 12)$

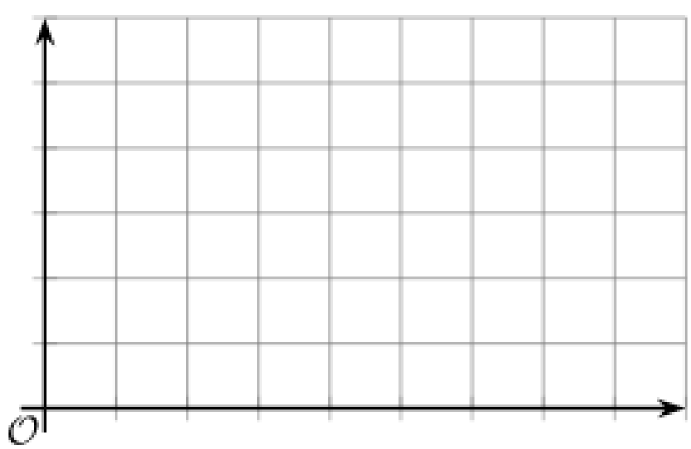
(From Unit 2)

10. Noah is baking a birthday cake for his dad's 50th birthday party. When he puts the cake in the oven at 2:00, the cake is 0.5 inches high in the pan. Noah checks the cake after 20 minutes to see how much it has risen, it is 1 inch high in the pan. Finally, Noah takes the cake out after another 10 minutes, and it is 1.25 inches high in the pan. If the cake rises at a constant rate as it bakes:

    a. Sketch a graph of the cake's height as a function of the time it baked.



    b. Sketch a graph of the time the cake baked as a function of its height.



(Addressing NC.8.F.5)

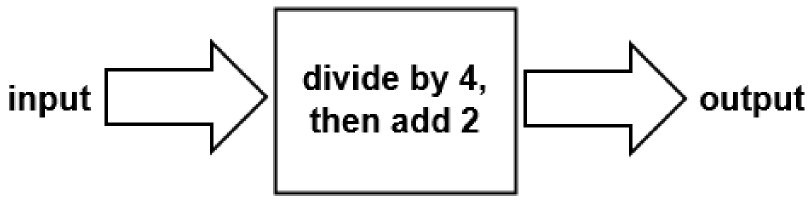
Lesson 4: Using Function Notation to Describe Rules (Part One)

Learning Targets

- I can make sense of rules of functions when they are written in function notation and create tables and graphs to represent the functions.
- I can write equations that represent the rules of functions.

Bridge

Given the function rule:[1]



Complete the table for the following input values:

Input	0	2	4	6	8	10
Output						

Warm-up: Two Functions

What do you notice? What do you wonder?

x	$f(x) = 10 - 2x$
1	8
1.5	7
5	0
-2	14

x	$g(x) = 3x^2$
-2	12
0	0
1	3
3	27

Activity 1: Four Functions

Here are descriptions and equations that represent four functions.

Descriptions	Equations
a. To get the output, subtract 7 from the input, then divide the result by 3.	$f(x) = 3x - 7$
b. To get the output, subtract 7 from the input, then multiply the result by 3.	$g(x) = 3(x - 7)$
c. To get the output, multiply the input by 3, then subtract 7 from the result.	$h(x) = \frac{x}{3} - 7$
d. To get the output, divide the input by 3, then subtract 7 from the result.	$k(x) = \frac{x-7}{3}$

1. Match each equation with a verbal description that represents the same function. Record your results.

   a. b.

   c. d.

2. For one of the functions, when the input is 6, the output is -3. Which is that function: f, g, h, or k? Explain how you know.

3. Which function value—$f(x)$, $g(x)$, $h(x)$, or $k(x)$—is the greatest when the input is 0? What about when the input is 10?

Are You Ready For More?

Mai says $f(x)$ is always greater than $g(x)$ for each value of x. Is this true? Explain how you know.

Activity 2: Rules for Area and Perimeter

1. A square that has a side length of 9 cm has an area of 81 cm^2. The relationship between the side length and the area of the square is a function.

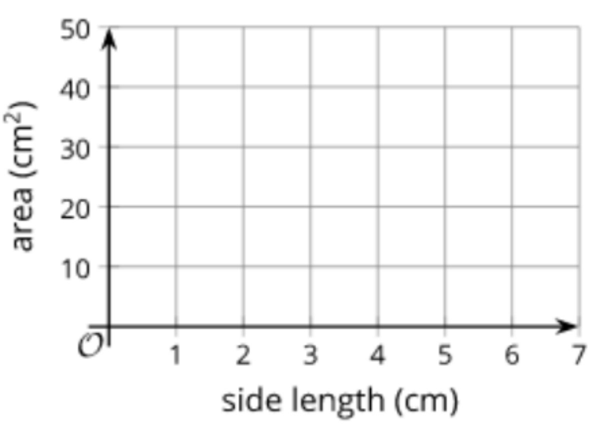
    a. Complete the table with the area for each given side length.

    Then, write a rule for a function A that gives the area of the square in cm^2 when the side length is s cm. Use function notation.

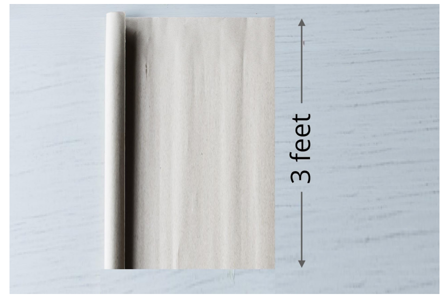
Side length (cm)	Area (cm^2)
1	
2	
4	
6	
s	

    b. What does $A(2)$ represent in this situation? What is its value?

    c. On the coordinate plane, sketch a graph of this function.



2. A roll of paper that is 3 feet wide can be cut to any length.



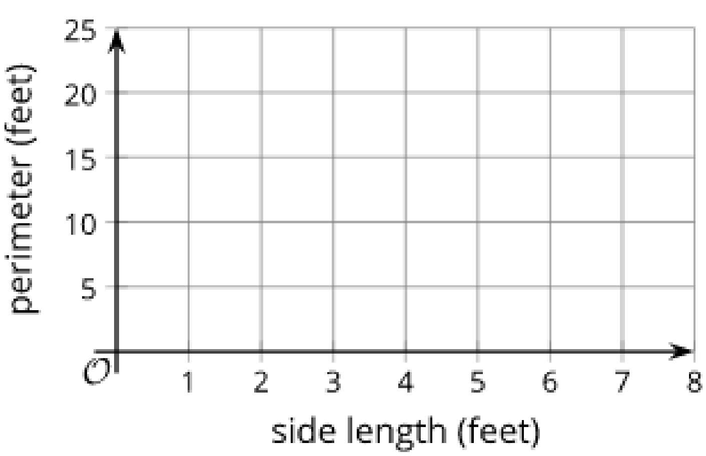
a. If we cut a length of 2.5 feet, what is the perimeter of the paper?

b. Complete the table with the perimeter for each given side length. Then, write a rule for a function P that gives the perimeter of the paper in feet when the side length in feet is l. Use function notation.

Side length (feet)	Perimeter (feet)
1	
2	
6.3	
11	
t	

c. What does $P(11)$ represent in this situation? What is its value?

d. On the coordinate plane, sketch a graph of this function.



Lesson Debrief

Lesson 4 Summary and Glossary

Some functions are defined by rules that specify how to compute the output from the input. These rules can be verbal descriptions or expressions and equations. For example:

Rules in words:

- To get the output of function f, add 2 to the input, then multiply the result by 5.
- To get the output of function m, multiply the input by $\frac{1}{2}$ and subtract the result from 3.

Rules in function notation:

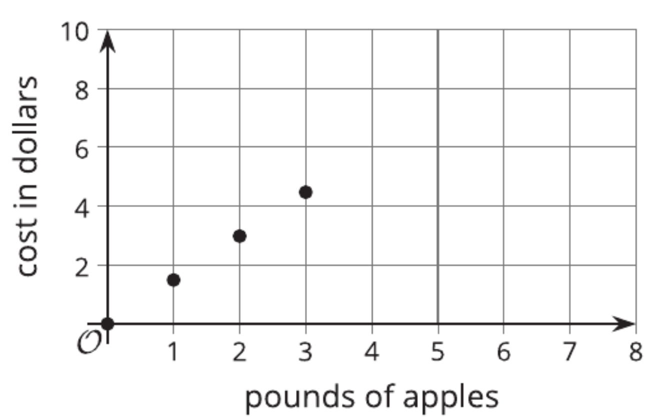
- $f(x) = (x+2) \cdot 5$ or $5(x+2)$
- $m(x) = 3 - \frac{1}{2}x$

Some functions that relate two quantities in a situation can also be defined by rules and can therefore be expressed algebraically, using function notation. Suppose function c gives the cost of buying n pounds of apples at $\$1.49$ per pound. We can write the rule $c(n) = 1.49n$ to define function c.

To see how the cost changes when n changes, we can create a table of values.

Pounds of applies, n	**Cost in dollars, $c(n) = 1.49n$**
0	0
1	1.49
2	2.98
3	4.47
n	1.49n

Plotting the pairs of values in the table gives us a graphical representation of c.



Unit 5 Lesson 4 Practice Problems

1. Match each equation with a description of the function it represents.

a. $f(x) = 2x + 4$

b. $g(x) = 2(x + 4)$

c. $h(x) = 4x + 2$

d. $k(x) = 4(x + 2)$

1. To get the output, add 4 to the input, then multiply the result by 2.
2. To get the output, add 2 to the input, then multiply the result by 4.
3. To get the output, multiply the input by 2, then add 4 to the result.
4. To get the output, multiply the input by 4, then add 2 to the result.

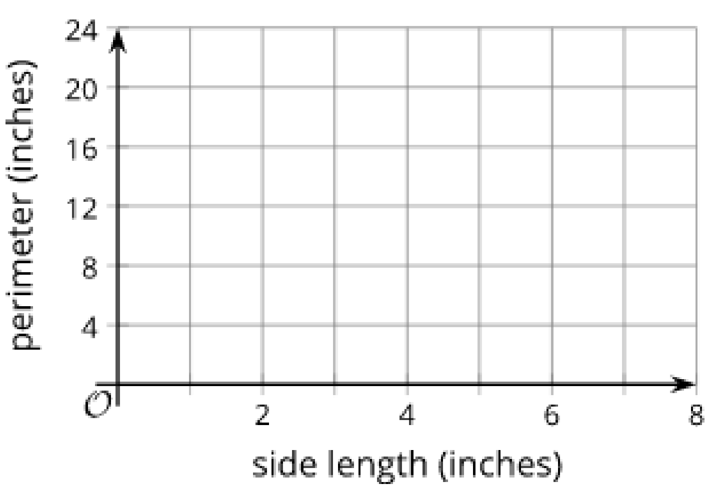
2. Function P represents the perimeter, in inches, of a square with side length x inches.

a. Complete the table.

x	0	1	2	3	4	5	6
$P(x)$							

b. Write an equation to represent function P.

c. Sketch a graph of function P.



3. Functions f and A are defined by these equations.

$$f(x) = 80 - 15x$$

$$A(x) = 25 + 10x$$

   a. Which function has a greater value when x is 2.5?

   b. Which function has a greater value when x is -2.5?

4. An equilateral triangle has three sides of equal length. Function P gives the perimeter of an equilateral triangle of side length s.

   a. Find $P(2)$

   b. Find $P(10)$

   c. Find $P(s)$

5. Function C gives the cost, in dollars, of buying n apples. Which statement best represents the meaning of $C(10) = 9$?

   a. The cost of buying 9 apples

   b. The cost of 9 apples is $10.

   c. The cost of 10 apples

   d. Ten apples cost $9.

(From Unit 5, Lesson 2)

6. Imagine a situation where a person is using a garden hose to fill a child's pool. Think of two quantities that are related in this situation and that can be seen as a function.

    a. Define the function using a statement in the form "_______ is a function of _______." Be sure to consider the units of measurement.

    b. Sketch a possible graph of the function. Be sure to label the axes.

    Then, identify the coordinates of one point on the graph and explain its meaning.

(From Unit 5, Lesson 1)

7. Diego is baking cookies for a fundraiser. He opens a 5-pound bag of flour and uses 1.5 pounds of flour to bake the cookies.

    Which equation or inequality represents f, the amount of flour left in the bag, after Diego bakes the cookies?

    a. $f = 1.5$

    b. $f < 1.5$

    c. $f = 3.5$

    d. $f > 3.5$

(From Unit 2)

8. The data set represents the number of cars in a town given a speeding ticket each day for 10 days.

2	4	5	5	7	7	8	8	8	12

a. What is the median? Interpret this value in the situation.

b. What is the IQR?

(From Unit 1)

9. Mai took a survey of students in her class to find out how many hours they spend reading each week. Here are some summary statistics for the data that Mai gathered:

- mean: 8.5 hours
- standard deviation: 5.3 hours
- median: 7 hours
- Q1: 5 hours
- Q3: 11 hours

a. Give an example of an outlier, and explain your reasoning.

b. Are there any outliers below the median? Explain your reasoning.

(From Unit 1)

Lesson 5: Using Function Notation to Describe Rules (Part Two)

Learning Targets

- I know different ways to find the value of a function and to solve equations written in function notation.
- I know what makes a function a linear function.
- I can use technology to graph a function given in function notation and use the graph to find the values of the function.

Warm-up: Make It True

Consider the equation $q = 4 + 0.8p$. Explain or show your reasoning.

1. What value of q would make the equation true when:

    a. p is 7?

    b. p is 100?

2. What value of p would make the equation true when:

    a. q is 12?

    b. q is 60?

Activity 1: Data Plans

A college student is choosing between two data plans for her new cell phone. Both plans include an allowance of 2 gigabytes of data per month. The monthly cost of each option can be seen as a function and represented with an equation:

- Option A: $A(x) = 60$
- Option B: $B(x) = 10x + 25$

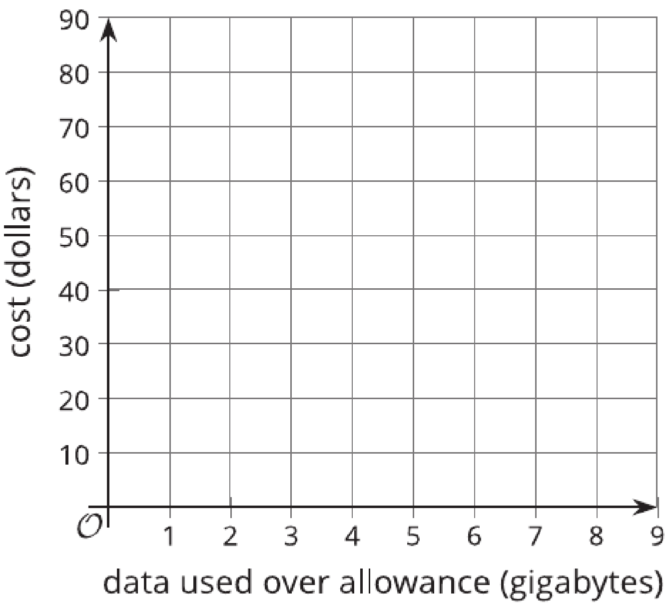
In each function, the input, x, represents the gigabytes of data used *over* the monthly allowance.

1. The student decides to find the values of $A(1)$ and $B(1)$ and compare them. What are those values?

2. After looking at some of her past phone bills, she decided to compare $A(7.5)$ and $B(7.5)$. What are those values?

3. Describe each data plan in words.

4. Graph each function on the same coordinate plane. Then, explain which plan you think she should choose.



5. The student only budgeted \$50 a month for her cell phone. She thought, “I wonder how many gigabytes of data I would have for \$50 if I go with Option B?” and wrote $B(x) = 50$. What is the answer to her question? Explain or show how you know.

Are You Ready For More?

Describe a different data plan that, for any amount of data used, would cost no more than one of the given plans and no less than the other given plan. Explain or show how you know this data plan would meet these requirements.

Activity 2: Function Notation and Graphing Technology

The function B is defined by the equation $B(x) = 10x + 25$. Use graphing technology to:

1. Find the value of each expression:
    a. $B(6)$
    b. $B(2.75)$
    c. $B(1.482)$
    d. $B(-3.5)$
2. Solve each equation:
    a. $B(x) = 93$
    b. $B(x) = 42.1$
    c. $B(x) = -78$
    d. $B(x) = 116.25$

Lesson Debrief

Lesson 5 Summary and Glossary

Knowing the rule that defines a function can be very useful. It can help us to:

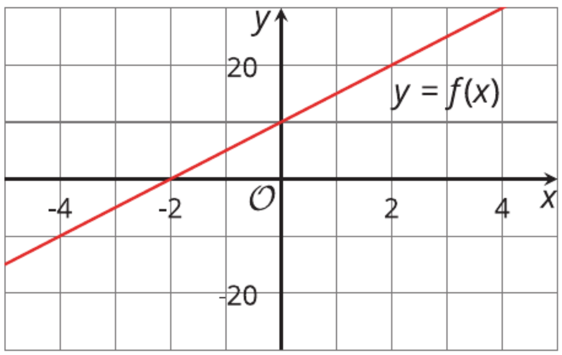
- Find the output when we know the input.
  - If the rule $f(x) = 5(x+2)$ defines f, we can find $f(100)$ by evaluating $5(100+2)$.
  - If $m(x) = 3 - \frac{1}{2}x$ defines function m, we can find $m(10)$ by evaluating $3 - \frac{1}{2}(10)$.
- Create a table of values.
- Here are tables representing functions f and m:

x	$f(x) = 5(x+2)$
-2	0
-1	5
0	10
1	15
2	20
3	25
4	30

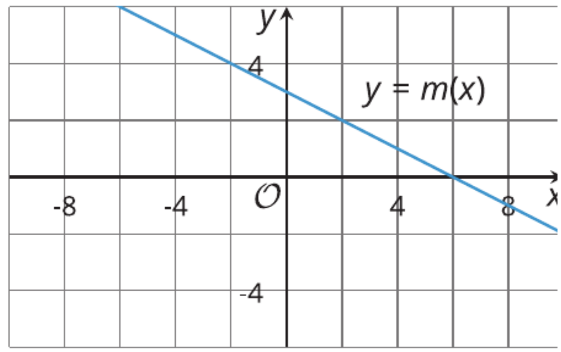
x	$m(x) = 3 - \frac{1}{2}x$
-2	4
-1	$3\frac{1}{2}$
0	3
1	$2\frac{1}{2}$
2	2
3	$1\frac{1}{2}$
4	1

- Graph the function. The horizontal values represent the input, and the vertical values represent the output.

For function f, the values of $f(x)$ are the vertical values, which are often labeled y, so we can write $y = f(x)$. Because $f(x)$ is defined by the expression $5(x+2)$, we can graph $y = 5(x+2)$.



For function m, we can write $y = m(x)$ and graph $y = 3 - \frac{1}{2}x$.



- Find the input when we know the output.
  - Suppose the output of function f is 65 at some value of x, or $f(x) = 65$, and we want to find out what that value is. Because $f(x)$ is equal to $5(x+2)$, we can write $5(x+2) = 65$ and solve for x.

$$5(x+2) = 65$$
$$x + 2 = 13$$
$$x = 11$$

Each function here is a **linear function** because the value of the function changes by a constant rate and its graph is a line.

Linear function: A function that has a constant rate of change. Another way to say this is that it grows by equal differences over equal intervals. For example, $f(x) = 4x - 3$ defines a linear function. Any time x increases by 1, $f(x)$ increases by 4.

Unit 5 Lesson 5 Practice Problems

1. The cell phone plan from company C costs $10 per month, plus $15 per gigabyte for data used. The plan from company D costs $80 per month, with unlimited data.

   Rule C gives the monthly cost, in dollars, of using g gigabytes of data on company C's plan. Rule D gives the monthly cost, in dollars, of using g gigabytes of data on company D's plan.

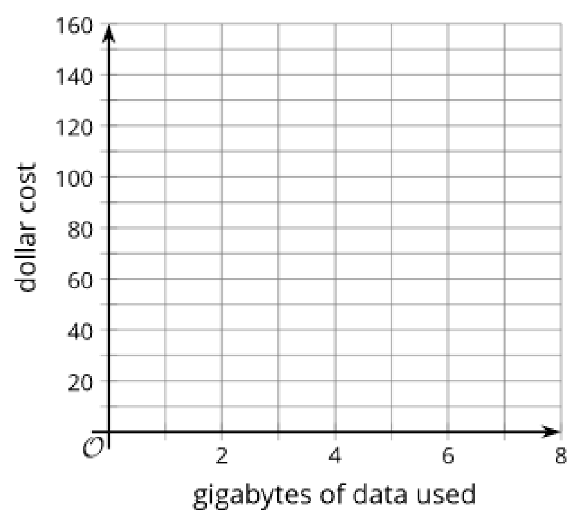
   a. Write a sentence describing the meaning of the statement $C(2) = 40$.

   b. Which value is smaller, $C(4)$ or $D(4)$? What does this mean for the two phone plans?

   c. Which value is smaller, $C(5)$ or $D(5)$? Explain how you know.

   d. For what number g is $C(g) = 130$?

   e. Draw the graph of each function.



2. Function g is represented by the graph.

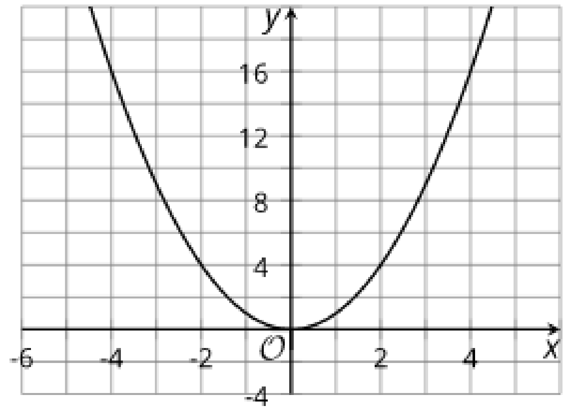
   For what input value or values is $g(x) = 4$?

   a. 2

   b. -2 and 2

   c. 16

   d. none



3. Function P gives the perimeter of an equilateral triangle of side length s. It is represented by the equation $P(s) = 3s$.

   a. What does $P(s) = 60$ mean in this situation?

   b. Find a value of s to make the equation $P(s) = 60$ true.

4. Function W gives the weight of a puppy, in pounds, as a function of its age, t, in months.

   Describe the meaning of each statement.

   a. $W(2) = 5$

   b. $W(6) > W(4)$

   c. $W(12) = W(15)$

(From Unit 5, Lesson 3)

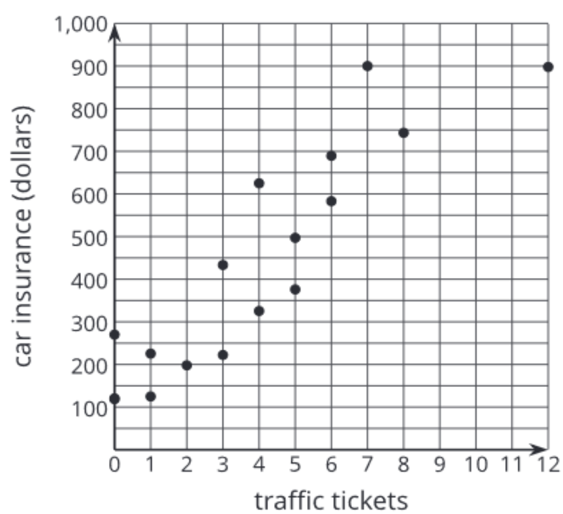
5. Function G takes a student's first name for its input and gives the number of letters in the first name for its output.

    a. Describe the meaning of $G(Jada) = 4$.

    b. Find the value of $G(Diego)$.

(From Unit 5, Lesson 2)

6. A recent survey investigated the relationship between the number of traffic tickets a person received and the cost of the person's car insurance. The scatter plot displays the relationship. The line that models the data is given by the equation $y = 73x + 146.53$, where x represents the number of traffic tickets, and y represents the cost of car insurance.



    a. The slope of the line is 73. What does this mean in this situation? Is it realistic?

    b. The y-intercept is $(0, 146.53)$. What does this mean in this situation?

(From Unit 4)

7. Diego is building a fence for a rectangular garden. It needs to be at least 10 feet wide and at least 8 feet long. The fencing he uses costs $3 per foot. His budget is $120. He wrote some inequalities to represent the constraints in this situation below.

   a. Explain what each equation or inequality represents.

Equation or inequalities	**What do they represent?**
$f = 2x + 2y$	
$x \geq 10$	
$y \geq 8$	
$3f \leq 120$	

   b. His mom says he should also include the inequality $f > 0$. Do you agree? Explain your reasoning.

(From Unit 3)

8. Members of the band sold juice and popcorn at a college football game to raise money for an upcoming trip. The band raised $2,000. The amount raised is divided equally among the m members of the band.

   Which equation represents the amount, A, each member receives?

   a. $A = \frac{m}{2000}$

   b. $A = \frac{2000}{m}$

   c. $A = 2000m$

   d. $A = 2000 - m$

(From Unit 2)

9. Answer the following questions:

   a. What is the five-number summary for 1, 3, 3, 3, 4, 8, 9, 10, 10, 17?

   b. When the maximum, 17, is removed from the data set, what is the five-number summary?

(From Unit 1)

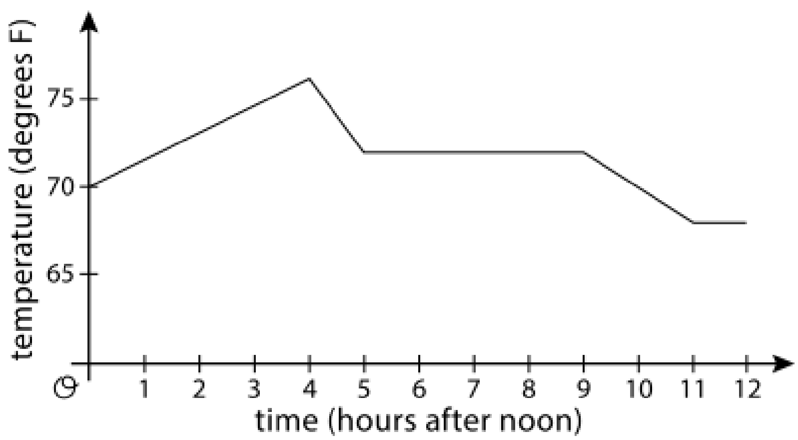
Lesson 6: Features of Graphs

Learning Targets

- I can identify important features of graphs of functions and explain what they mean in the situations represented.
- I understand and can use the terms “horizontal intercept,” “vertical intercept,” “maximum,” and “minimum” when talking about functions and their graphs.

Bridge

This graph shows the temperature in Diego’s house between noon and midnight one day.[1]



Select **all** the true statements.

a. Time is a function of temperature.

b. The lowest temperature occurred between 4:00 and 5:00.

c. The temperature was increasing between 9:00 and 10:00.

d. The temperature was 74 degrees twice during the 12-hour period.

e. There was a 4-hour period during which the temperature did not change.

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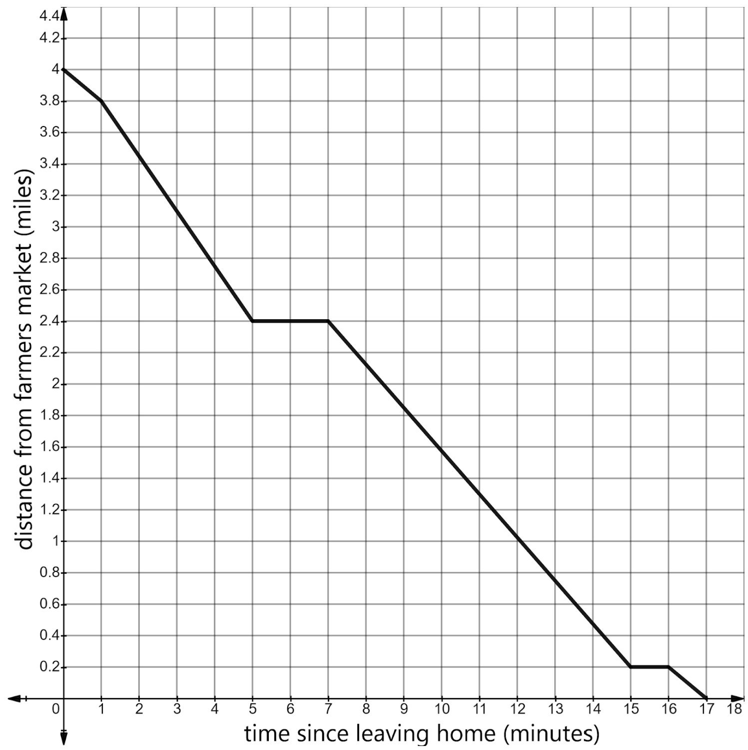
[1] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

Warm-up: Farmers Market

Noah and a sibling are going to make their favorite dinner. To find the ingredients, they take a Lyft ride from their home to Rosa Parks Farmers Market. This graph represents function d, which gives his distance from the farmers market, in miles, m minutes since leaving his home.



Use the graph to find or estimate:

1. $d(0)$

2. $d(12)$

3. the value of m when $d(m) = 2.4$

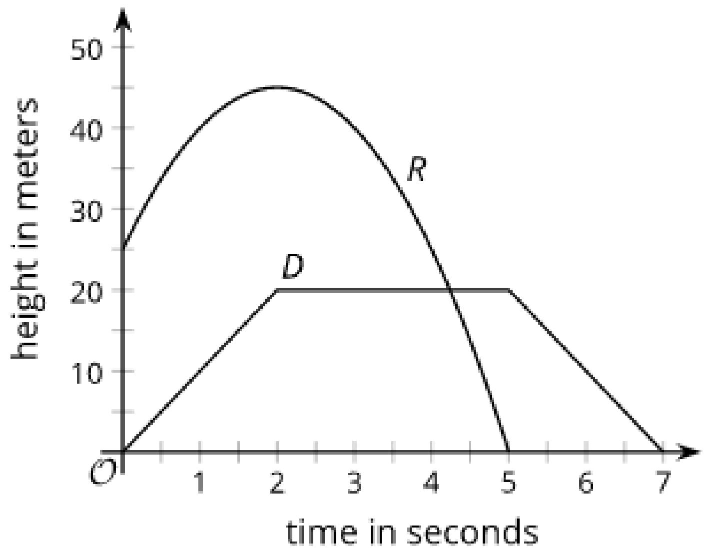
4. the value of m when $d(m) = 0$

Activity 1: A Toy Rocket and a Drone

A toy rocket and a drone were launched at the same time.

Here are the graphs that represent the heights of two objects as a function of time since they were launched.

Height is measured in meters above the ground and time is measured in seconds since launch.



1. Analyze the graphs and describe—as precisely as you can—what was happening with each object. Your descriptions should be complete and precise enough that someone who is not looking at the graph could visualize how the objects were behaving.

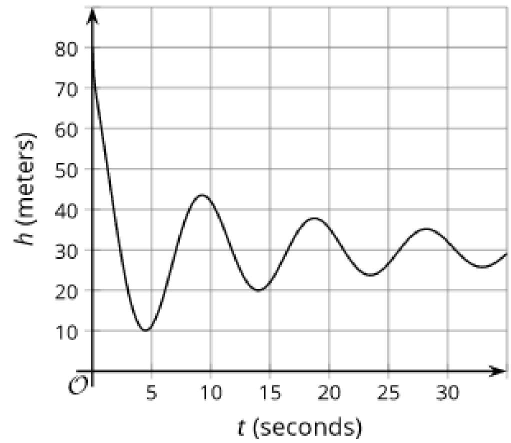
2. Which parts or features of the graphs show important information about each object's movement? List the features or mark them on the graphs.

Activity 2: The Jump

In a bungee jump, the height of the jumper is a function of time since the jump begins.

Function h defines the height, in meters, of a jumper above a river, t seconds since leaving the platform.Here is a graph of function h, followed by five expressions or equations and five graphical features.

Expressions or equations	Features of graph
• $h(0)$	• First dip in the graph
• $h(t) = 0$	• Vertical intercept
• $h(4)$	• First peak in the graph
• $h(t) = 80$	• Horizontal intercept
• $h(t) = 45$	• Maximum



Match each description of the jump to a corresponding expression or equation and to a feature on the graph. One expression or equation does not have a matching verbal description. Its corresponding graphical feature is also not shown on the graph. Interpret that expression or equation in terms of the jump and in terms of the graph of the function. Record your interpretation in the last row of the table.

Description of jump	Expression or equation	Feature of graph
a. The greatest height that the jumper is from the river		
b. The height from which the jumper was jumping		
c. The time at which the jumper reached the highest point after the first bounce		
d. The lowest point that the jumper reached in the entire jump		
e. ____________________		

Are You Ready For More?

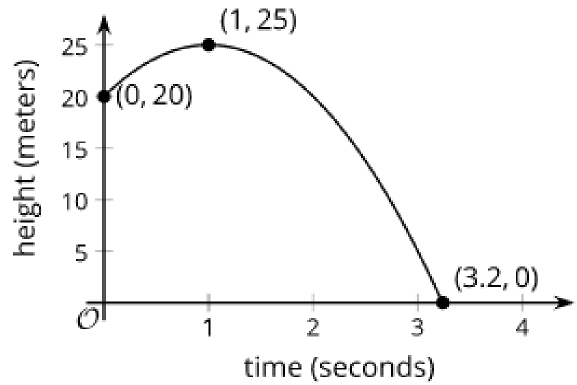
Based on the information available, how long do you think the bungee cord is? Make an estimate and explain your reasoning.

Lesson Debrief

Lesson 6 Summary and Glossary

The graph of the function can give us useful information about the quantities in a situation. Some points and features of a graph are particularly informative, so we pay closer attention to them.

Let's look at the graph of function h, which gives the height, in meters, of a ball t seconds after it is tossed up in the air. From the graph, we can see that:



- The point $(0, 20)$ is the **vertical intercept** of the graph, or the point where the graph intersects the vertical axis.

  This point tells us that the initial height of the ball is 20 meters, because when t is 0, the value of $h(t)$ is 20. The statement $h(0) = 20$ captures this information.

> **Vertical intercept**: The point where a graph crosses the vertical axis, so its coordinates have the form $(0, b)$. In the graph of a function, b represents the output for an input of 0. If the axis is labeled with the variable y, the vertical intercept is also called the y-intercept.
>
> The term is sometimes used to mean just the y-coordinate of the point where the graph crosses the vertical axis. The vertical intercept of the graph of $y = 3x - 5$ is $(0, -5)$, or just -5.

- The point $(1, 25)$ is the highest point on the graph, so it is a **maximum** of the graph. The value 25 is also the maximum value of the function h. It tells us that the highest point the ball reaches is 25 feet, and that this happens 1 second after the ball is tossed.

> **Maximum**: A value of the function that is greater than or equal to all the other values. The maximum of the graph of the function is the corresponding highest point on the graph.

- The point $(3.2, 0)$ is a **horizontal intercept** of the graph, a point where the graph intersects the horizontal axis. This point is also the lowest point on the graph, so it represents a **minimum** of the graph. This tells us that the ball hits the ground 3.2 seconds after being tossed up, so the height of the ball is 0 when t is 3.2, which we can write as $h(3.2) = 0$. Because h cannot have any lower value, 0 is also the minimum value of the function.

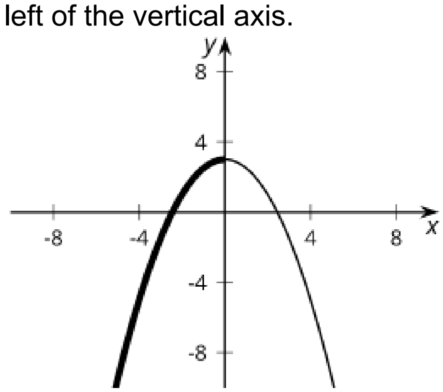
> **Horizontal intercept**: The point where a graph crosses the horizontal axis, so its coordinates have the form $(a, 0)$. In the graph of a function, a is the input value that results in an output of 0. If the axis is labeled with the variable x, the horizontal intercept is also called the x-intercept. The term is sometimes used to refer only to the x-coordinate of the point where the graph crosses the horizontal axis.

Minimum: A value of the function that is less than or equal to all the other values. The minimum of the graph of the function is the corresponding lowest point on the graph.

- The height of the graph increases when t is between 0 and 1. Then, the graph changes direction and the height decreases when t is between 1 and 3.2. Neither the **increasing** part nor the **decreasing** part is a straight line. This means that the ball increases in height in the first second after being tossed, and then falls between 1 second and 3.2 seconds. It also tells us that the height does not increase or decrease at a constant rate.

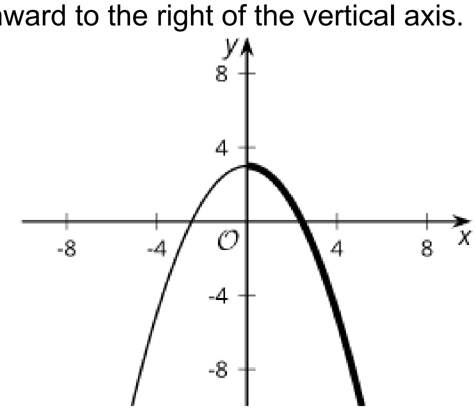
Increasing (function): A function is increasing if its outputs get larger as the inputs get larger, resulting in an upward sloping graph as you move from left to right.

A function can also be increasing just for a certain interval. For example the function f given by $f(x) = 3 - x^2$, whose graph is shown, is increasing for $x \leq 0$ because the graph slopes upward to the left of the vertical axis.



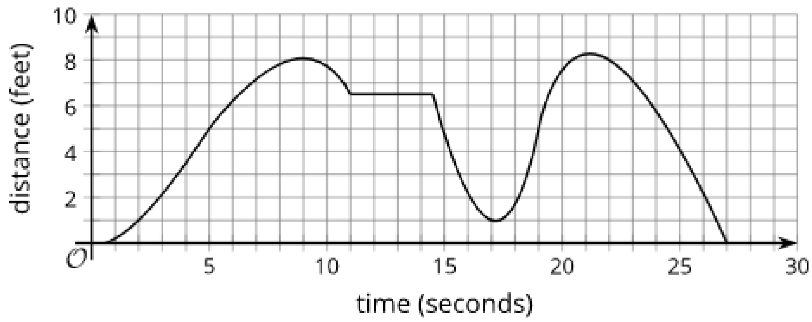
Decreasing (function): A function is decreasing if its outputs get smaller as the inputs get larger, resulting in a downward sloping graph as you move from left to right.

A function can also be decreasing just for a certain interval. For example the function f given by $f(x) = 3 - x^2$, whose graph is shown, is decreasing for $x \geq 0$ because the graph slopes downward to the right of the vertical axis.



Unit 5 Lesson 6 Practice Problems

1. This graph represents Andre's distance from his bicycle as he walks in a park.



Decide whether the following statements are **true** or **false**.

a. The graph has multiple horizontal intercepts.

b. A horizontal intercept of the graph represents the time when Andre was with his bike.

c. A minimum of the graph is $(17, 1)$.

d. The graph has two maximums.

e. About 21 seconds after he left his bike, he was the farthest away from it, at about 8.3 feet.

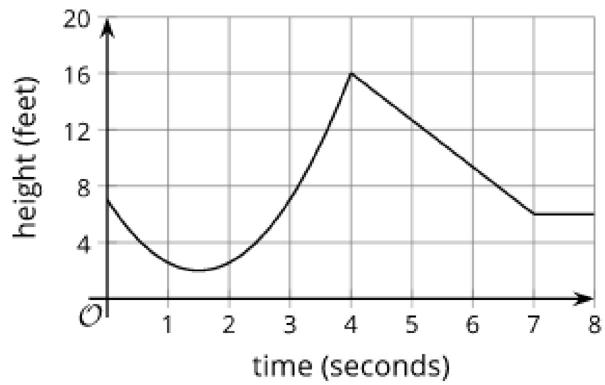
2. The graph represents the temperature in degrees Fahrenheit as a function of time.

a. Tell the story of the temperature throughout the day.

b. Identify the maximum and minimum of the function and where the function is increasing and decreasing.

3. Match each feature of the situation with a corresponding statement in function notation.

a. maximum height b. minimum height c. height staying the same d. starting height	i. $h(0) = 7$ ii. $h(1.5)$ iii. $h(4)$ iv. $h(t) = 6$ for $7 \leq t \leq 8$



4. Here are the equations that define three functions.

$f(x) = 4x - 5$
$g(x) = 4(x - 5)$
$h(x) = \frac{x}{4} - 5$

a. Which function value is the largest: $f(100)$, $g(100)$, or $h(100)$?

b. Which function value is the largest: $f(-100)$, $g(-100)$, or $h(-100)$?

c. Which function value is the largest: $f(\frac{1}{100})$, $g(\frac{1}{100})$, or $h(\frac{1}{100})$?

(From Unit 5, Lesson 4)

5. Function f is defined by the equation $f(x) = x^2$.

   a. What is $f(2)$?

   b. What is $f(3)$?

   c. Explain why $f(2) + f(3) \neq f(5)$.

(From Unit 5, Lesson 4)

6. A sports journalist is trying to rank the eight best men's college basketball teams from the 2020–21 season in the Atlantic Coast Conference based on the number of games each team won last year against opponents in the conference. The table shows the colleges for several different win totals.

Games won	13	11	9	11	10	10	8	9
College team	Virginia	Florida State	Virginia Tech	Georgia Tech	Clemson	North Carolina	Louisville	Syracuse

   a. What does the independent variable in the relationship represent?

   b. What does the dependent variable in the relationship represent?

   c. Is the relationship a function? Why or why not?

(From Unit 5, Lesson 1)

7. On July 7, 2021, the hourly temperatures in Charlotte, in degrees Fahrenheit, were:

Hour	7 a.m.	8 a.m.	9 a.m.	10 a.m.	11 a.m.	12 p.m.	1 p.m.	2 p.m.	3 p.m.
Temperature	71	75	78	82	85	87	88	89	90

a. Use technology to determine the line of best fit. Use $x = 0$ to represent 7 a.m.

b. Elena says that at 10:00 p.m., the temperature should be about 104 degrees. Do you agree with Elena? Why or why not?

(From Unit 4)

8. Find the equation for a line perpendicular to $y = 3x - 7$ that passes through the origin.

(From Unit 3)

9. Priya bought two plants for a science experiment. When she brought them home, the first plant was 5 cm tall, and the second plant was 4 cm. Since then, the first plant has grown 0.5 cm a week, and the second plant has grown 0.75 cm a week.

   a. Which plant is taller at the end of 2 weeks? Explain your reasoning.

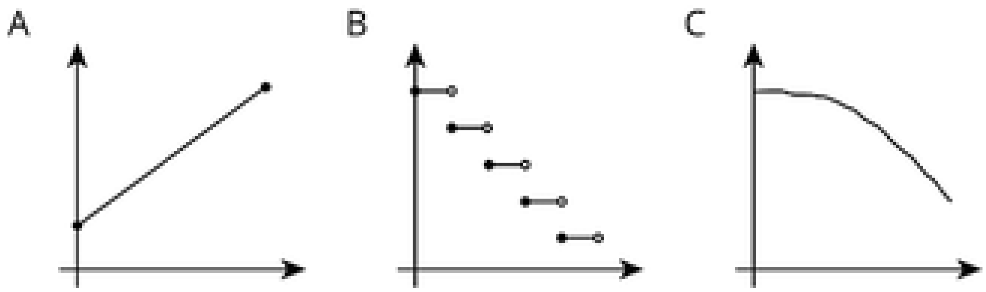
   b. Which plant is taller at the end of 10 weeks? Explain your reasoning.

   c. Priya represents this situation with the equation $5 + 0.5w = 4 + 0.75w$, where w represents the end of week. What does the solution to this equation, $w = 4$, represent in this situation?

   d. What does the solution to the inequality $5 + 0.5w > 4 + 0.75w$ represent in this situation?

(From Unit 2)

10. Match the graphs to the following situations (you can use a graph multiple times). For each match, name possible independent and dependent variables and how you would label the axes.[2]



a. Tyler pours the same amount of milk from a bottle every morning.

b. A plant grows the same amount every week.

c. The day started very warm but then it got colder.

d. A carnival has an entry fee of $5, and tickets for rides cost $1 each.

(Addressing NC.8.F.5)

[2] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

Lesson 7: Using Graphs to Find Average Rate of Change

Learning Targets

- When given a graph of a function, I can estimate or calculate the average rate of change between two points.
- I understand the meaning of the term "average rate of change."

Bridge

A function assigns to the inputs shown the corresponding outputs given in the table.[1]

Input	Output
1	2
2	-1
4	-7
6	-13

1. Do you suspect the function is linear? Compute the rate of change of this data for at least three pairs of inputs and their corresponding outputs.

2. What equation seems to describe the function?

3. As you did not verify that the rate of change is constant across **all** input/ output pairs, check that the equation you found in problem 1 does indeed produce the correct output for each of the four inputs 1, 2, 4, and 6.

4. What will the graph of the function look like? Explain.

[1] Adapted from EngageNY https://www.engageny.org/ for the New York State Department of Education, which was originally developed and authored by Great Minds. It is licensed under the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 United States (CC BY-NC-SA 3.0 US).

Warm-up: Temperature Drop

Here are the recorded temperatures at three different times on a winter evening.

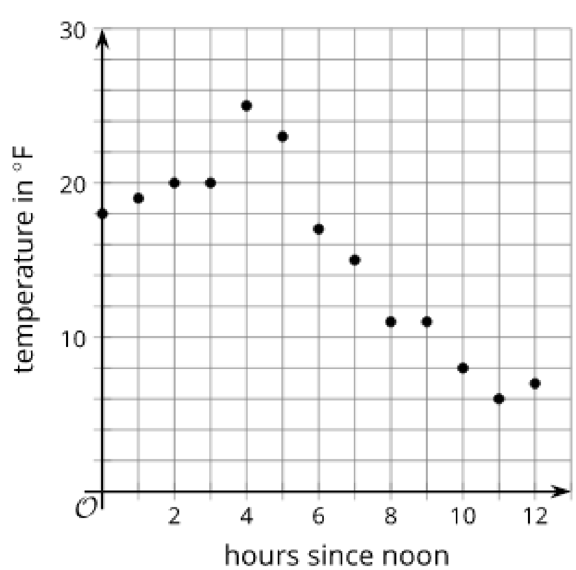
Time	4 p.m.	6 p.m.	10 p.m.
Temperature	$25°F$	$17°F$	$8°F$

- Tyler says the temperature dropped faster between 4 p.m. and 6 p.m.
- Mai says the temperature dropped faster between 6 p.m. and 10 p.m.

Who do you agree with? Explain your reasoning.

Activity 1: Drop Some More

The table and graphs show a more complete picture of the temperature changes on the same winter day. The function T gives the temperature in degrees Fahrenheit, h hours since noon.



h	$T(h)$
0	18
1	19
2	20
3	20
4	25
5	23
6	17
7	15
8	11
9	11
10	8
11	6
12	7

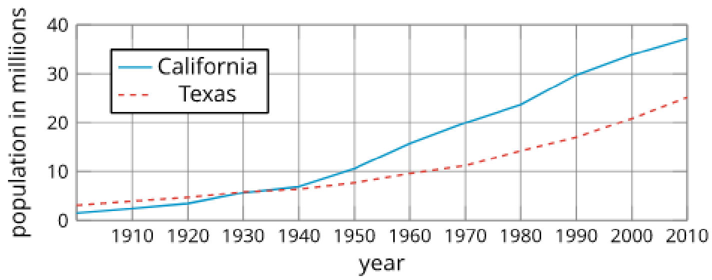
1. Find the average rate of change for the following intervals. Explain or show your reasoning.
    a. between noon and 1 p.m.
    b. between noon and 4 p.m.
    c. between noon and midnight
2. Remember Mai and Tyler's disagreement? Use average rate of change to show which time period—4 p.m. to 6 p.m. or 6 p.m. to 10 p.m.—experienced a faster temperature drop.

Are You Ready For More?

1. Over what interval did the temperature decrease the most rapidly?
2. Over what interval did the temperature increase the most rapidly?

Activity 2: Populations of Two States

The graphs show the populations of California and Texas over time.



1.
   a. Estimate the average rate of change in the population in each state between 1970 and 2010. Show your reasoning.

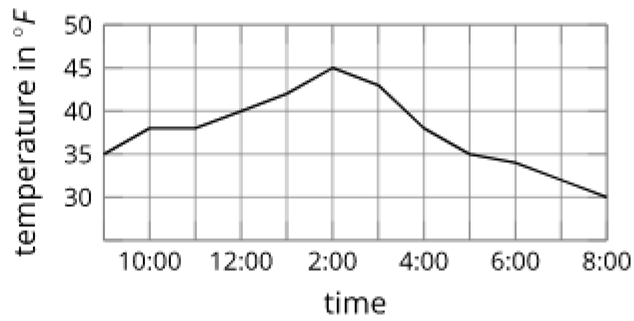
   b. In this situation, what does each rate of change mean?

2. Which state's population grew more quickly between 1900 and 2000? Show your reasoning.

Lesson Debrief

Lesson 7 Summary and Glossary

Here is a graph of one day's temperature as a function of time.



The temperature was $35°F$ at 9 a.m. and $45°F$ at 2 p.m., an increase of $10°F$ over those 5 hours.

The increase wasn't constant, however. The temperature rose from 9 a.m. and 10 a.m., stayed steady for an hour, then rose again.

- On average, how fast was the temperature rising between 9 a.m. and 2 p.m.?

  Let's calculate the **average rate of change** and measure the temperature change per hour. We do that by finding the difference in the temperature between 9 a.m. and 2 p.m. and dividing it by the number of hours in that interval.

  average rate of change $= \frac{45-35}{5} = \frac{10}{5} = 2$

  On average, the temperature between 9 a.m. and 2 p.m. increased $2°F$ per hour.

- How quickly was the temperature falling between 2 p.m. and 8 p.m.?

  average rate of change $= \frac{30-45}{6} = \frac{-15}{6} = -2.5$

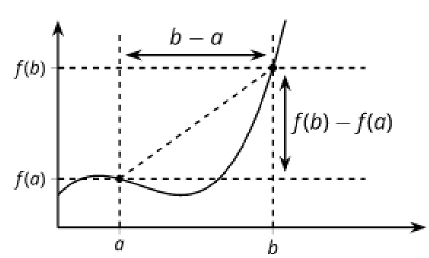
  On average, the temperature between 2 p.m. and 8 p.m. dropped by $2.5°F$ per hour.

In general, we can calculate the average rate of change of a function f, between input values a and b, by dividing the difference in the outputs by the difference in the inputs.

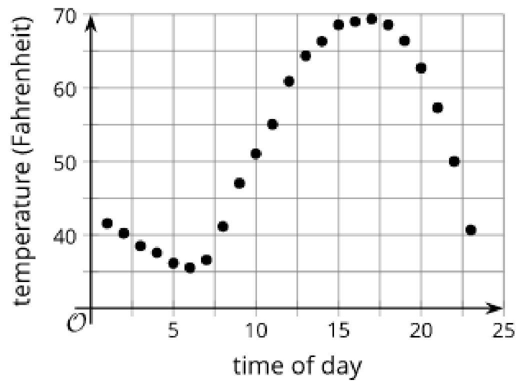
average rate of change $= \frac{f(b)-f(a)}{b-a}$

If the two points on the graph of the function are $(a, f(a))$ and $(b, f(b))$, the average rate of change is the slope of the line that connects the two points.

Average rate of change: The average rate of change of a function f between inputs a and b is the change in the outputs divided by the change in the inputs: $\frac{f(b)-f(a)}{b-a}$. It is the slope of the line joining $(a, f(a))$ and $(b, f(b))$ on the graph.



Unit 5 Lesson 7 Practice Problems



1. The temperature was recorded at several times during the day. Function T gives the temperature in degrees Fahrenheit, n hours since midnight. Here is a graph for this function.

   For each time interval, decide if the average rate of change is positive, negative, or zero.

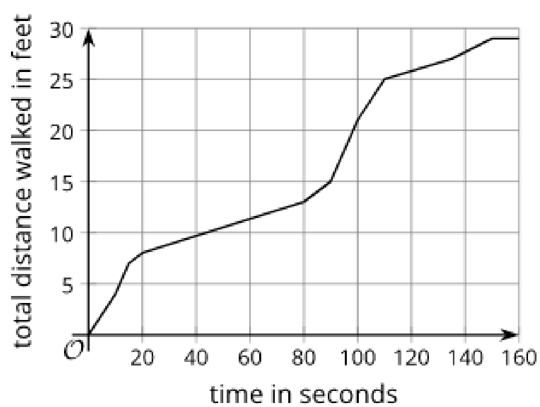
   a. From $n = 1$ to $n = 5$

   b. From $n = 5$ to $n = 7$

   c. From $n = 10$ to $n = 20$

   d. From $n = 15$ to $n = 18$

   e. From $n = 20$ to $n = 24$



2. The graph shows the total distance, in feet, walked by a person as a function of time, in seconds.

   a. Was the person walking faster between 20 and 40 seconds or between 80 and 100 seconds?

   b. Was the person walking faster between 0 and 40 seconds or between 40 and 100 seconds?

3. The height, in feet, of a squirrel running up and down a tree is a function of time, in seconds.

   Here are statements describing the squirrel's movement during four intervals of time. Match each description with a statement about the average rate of change of the function for that interval.

a. The squirrel runs up the tree very fast. b. The squirrel starts and ends at the same height. c. The squirrel runs down the tree. d. The squirrel runs up the tree slowly.	i. The average rate of change is negative. ii. The average rate of change is zero. iii. The average rate of change is small and positive. iv. The average rate of change is large and positive.

4. The percent of voters between the ages of 18 and 29 who participated in each United States presidential election between the years 1988 to 2016 are shown in the table.

Year	1988	1992	1996	2000	2004	2008	2012	2016
Percentage of voters ages 18–29	35.7	42.7	33.1	34.5	45.0	48.4	40.9	43.4

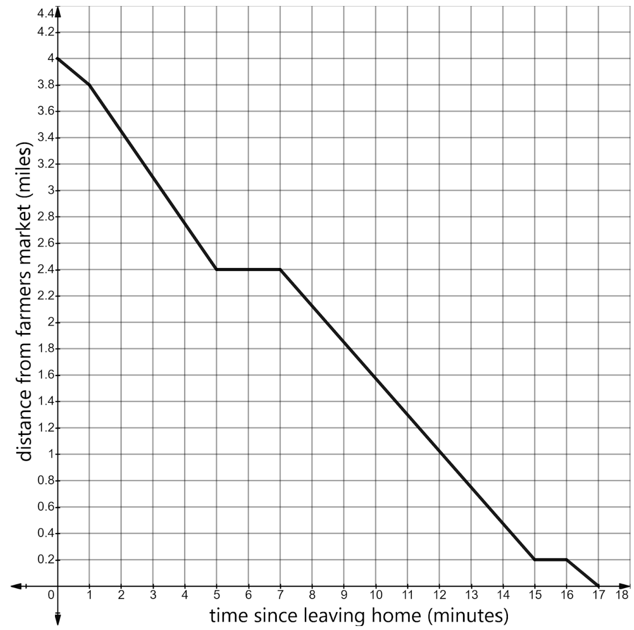
   The function P gives the percent of voters between 18 and 29 years old who participated in the election in year t.

   a. Determine the average rate of change for P between 1992 and 2000.

   b. Pick two different values of t so that the function has a negative average rate of change between the two values. Determine the average rate of change.

   c. Pick two values of t so that the function has a positive average rate of change between the two values. Determine the average rate of change.

5. Noah and a sibling are going to make their favorite dinner. To find the ingredients, they take a Lyft ride from their home to Rosa Parks Farmers Market. This graph represents function d, which gives his distance from the farmers market, in miles, m minutes since leaving his home.



   a. Estimate the average rate of change over Noah's entire route. Interpret this quantity for the context of the situation.

   b. What is the average rate of change from $m = 5$ to $m = 7$ and from $m = 15$ to $m = 16$? Interpret this quantity for the context of the situation and provide a possible explanation for this average rate of change.

6. Jada walks to school. The function D gives her distance from school, in meters, as a function of time, in minutes, since she left home.

   What does $D(10) = 0$ represent in this situation?

(From Unit 5, Lesson 2)

7. Jada walks to school. The function D gives her distance from school, in meters, t minutes since she left home.

   Which equation tells us, "Jada is 600 meters from school after 5 minutes"?

   a. $D(5) = 600$

   b. $D(600) = 5$

   c. $t(5) = 600$

   d. $t(600) = 5$

   (From Unit 5, Lesson 2)

8. A news website shows a scatter plot with a positive relationship between the number of vending machines in a school and the average percentage of students who are absent from school each day. The headline reads, "Vending machines are causing our youth to miss school!"

   a. What is wrong with this claim?

   b. What is a better headline for this information?

   (From Unit 4)

9. Select **all** numbers that are solutions to the inequality: $-\frac{1}{2}x - 8 < 4x + 5\frac{1}{2}$

   a. 5

   b. -5

   c. 3

   d. -3

   e. 0

   (From Unit 2)

10. An airplane needs to begin its descent to land at Charlotte Douglas International Airport. The equation $h = 27000 - 1800m$ represents the height, h, in feet of the airplane m minutes after beginning its descent.

    Identify the slope, horizontal intercept, and vertical intercept of the graph of this equation, and explain their meaning in the context of the airplane's descent.

    (Addressing NC.8.F.4)

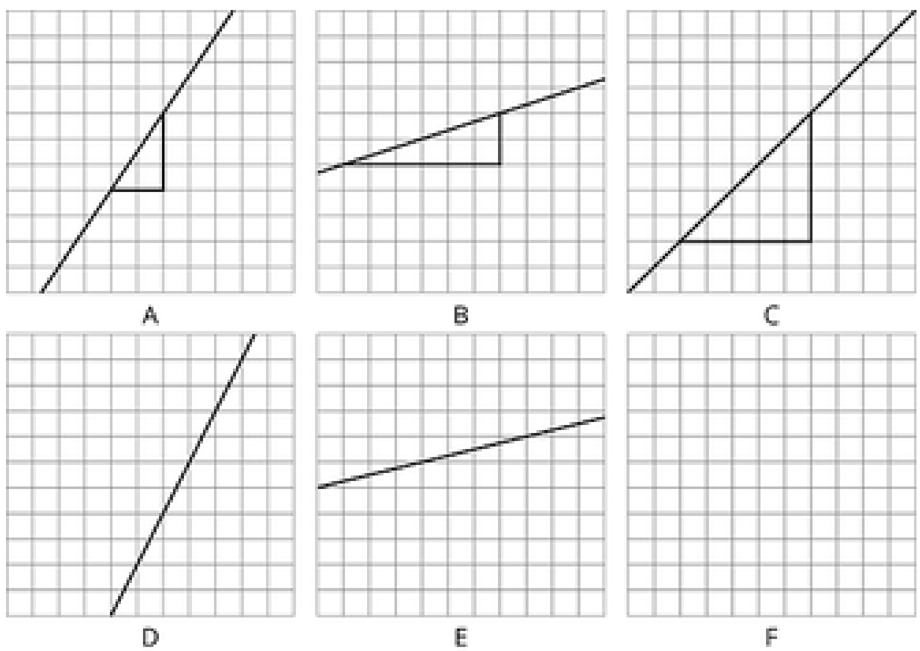
Lesson 8: Interpreting and Creating Graphs

Learning Targets

- When given a description or a visual representation of a situation, I can sketch a graph that shows important features of the situation.
- I can explain the average rate of change of a function in terms of a situation.
- I can make sense of important features of a graph and explain what they mean in a situation.

Bridge

Here are several lines:[1]



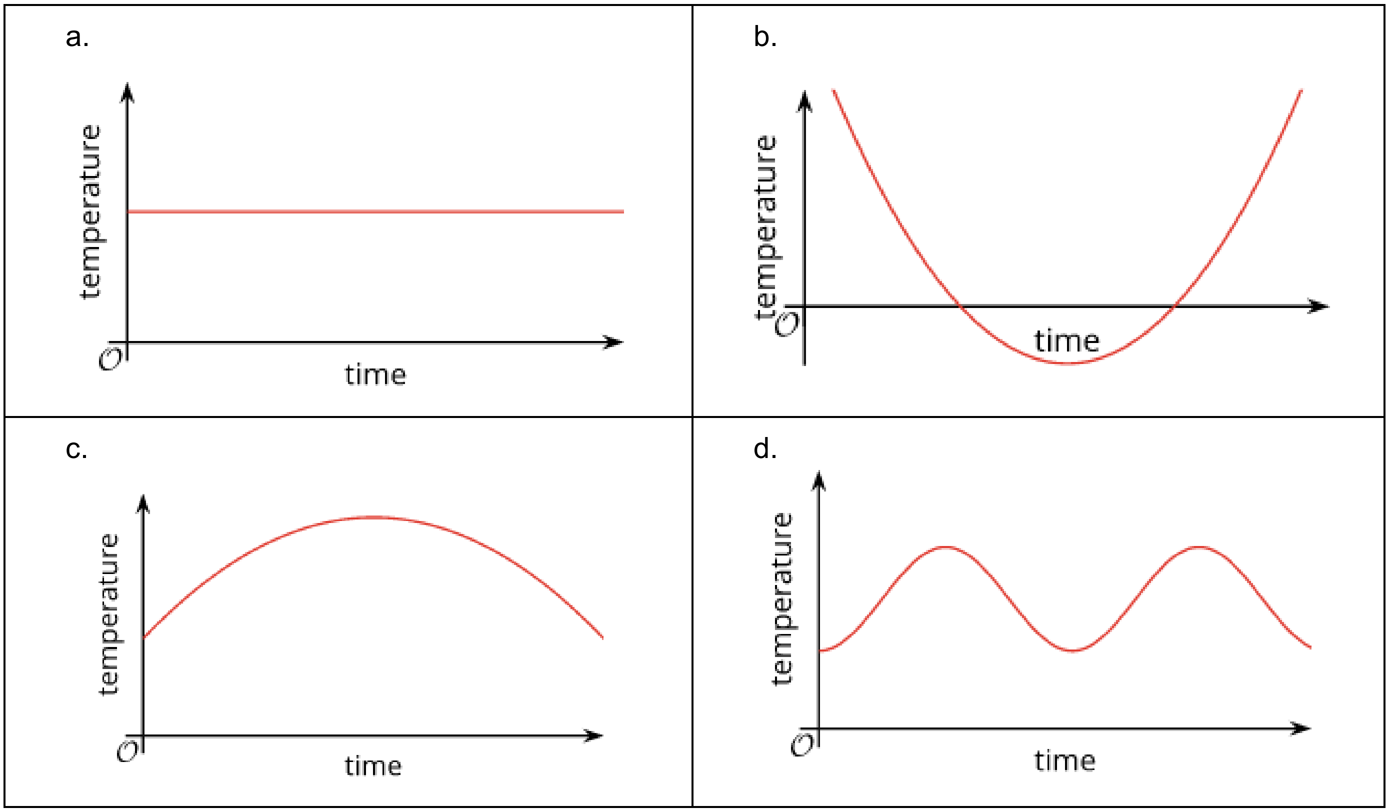
1. Match each line shown with a slope from this list: $\frac{1}{3}$, 2, 1, 0.25, $\frac{1}{2}$, $\frac{3}{2}$

2. One of the given slopes does not have a line to match. Draw a line with this slope on the empty grid (F).

[1] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

Warm-up: Temperature Over Time

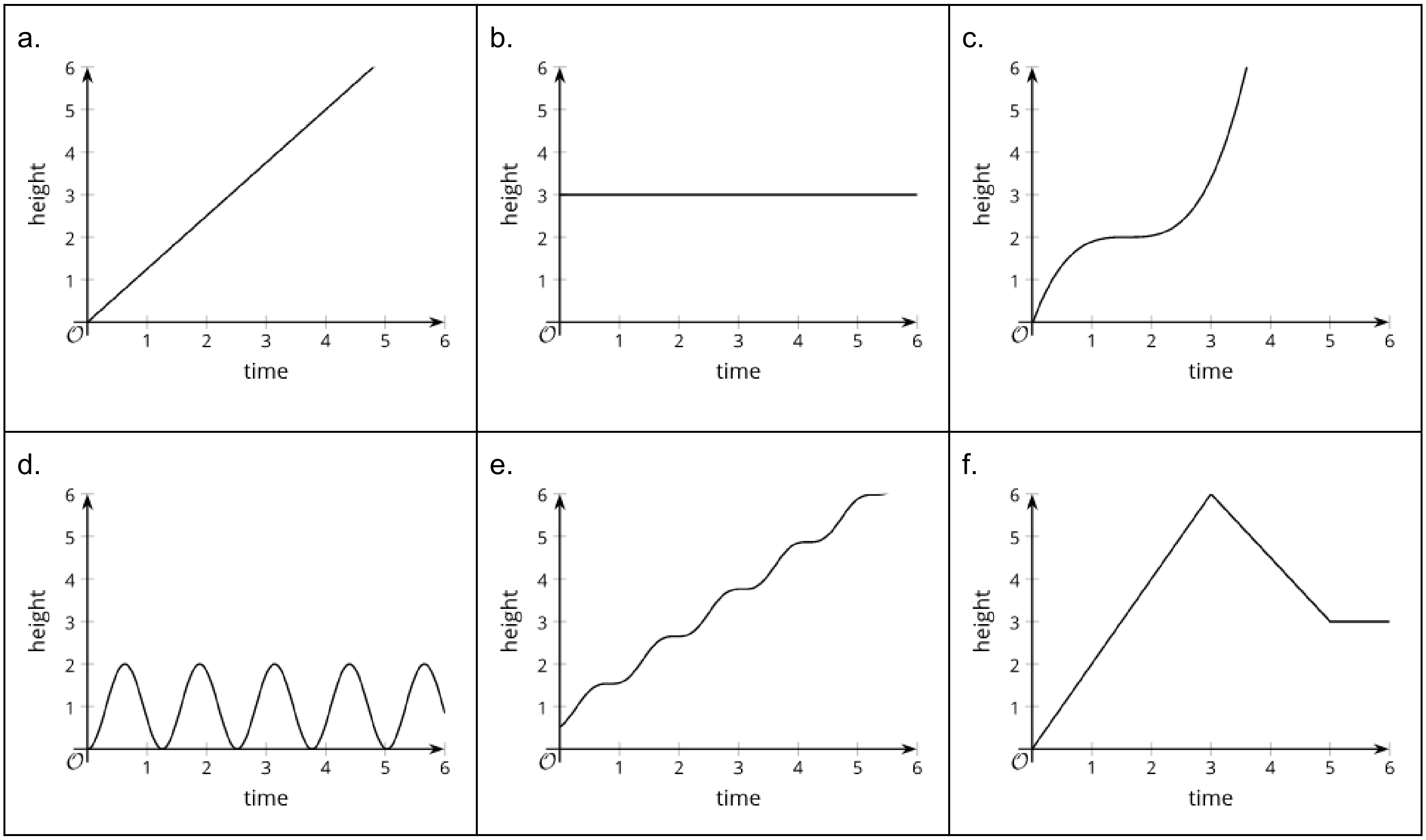
Which graph doesn't belong? Explain your reasoning.



Activity 1: Flag Raising (Part One)

A flag ceremony is held at a Fourth of July event. The height of the flag is a function of time.

Here are some graphs that could each be a possible representation of the function.

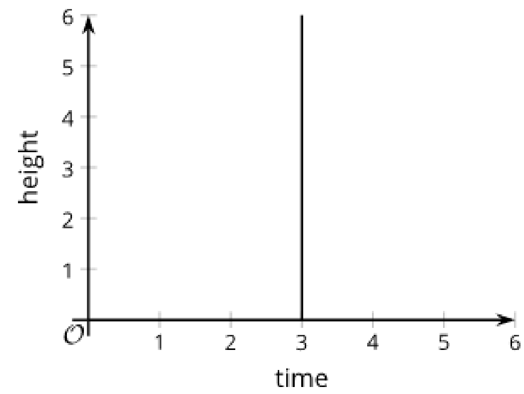


1.

   a. For each graph assigned to you, explain what it tells us about the flag.

   b. Decide as a group which graph(s) appear to be most realistic and which ones are least realistic.

2. Here is another graph that relates time and height.



   a. Can this graph represent the time and height of the flag? Explain your reasoning.

   b. Is this a graph of a function? Explain your reasoning.

Are You Ready For More?

Suppose an ant is moving at a rate of 1 millimeter per second and keeps going at that rate for a long time.

If time, x, is measured in seconds, then the distance the ant has traveled in millimeters, y, is $y = 1x$. If time, x, is measured in minutes, the distance in millimeters is $y = 60x$.

1. Explain why the equation $y = (365 \cdot 24 \cdot 3,600)x$ gives the distance the ant has traveled, in millimeters, as a function of time, x, in years.

2. Use graphing technology to graph the equation.

    a. Label the axes with appropriate quantities and units.

    b. Does the graph look like that of a function? Why do you think it looks this way?

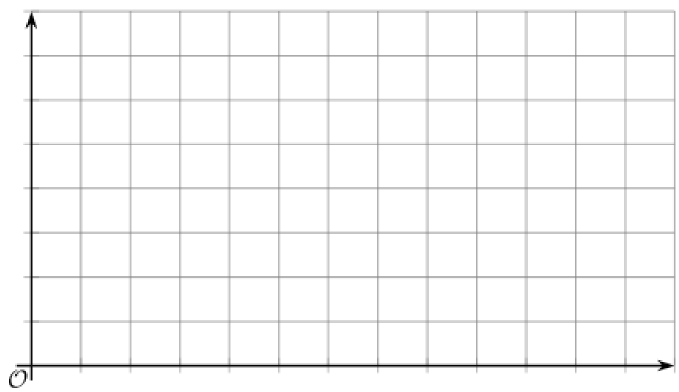
3. Adjust the graphing window until the graph no longer looks this way. If you manage to do so, describe the graphing window that you use.

4. Do you think the last graph in the flag activity could represent a function relating time and height of flag? Explain your reasoning.

Activity 2: Flag Raising (Part Two)

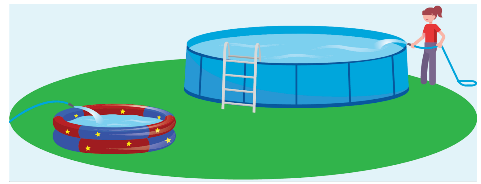
Your teacher will show a video of a flag being raised. Function H gives the height of the flag over time. Height is measured in feet. Time is measured in seconds since the flag is fully secured to the string, which is when the video clip begins.

1. On the coordinate plane, sketch a graph that could represent function H. Be sure to include a label and a scale for each axis.



2. Use your graph to estimate the average rate of change from the time the flag starts moving to the time it stops. Be prepared to explain what the average rate of change tells us about the flag.

Activity 3: Two Pools

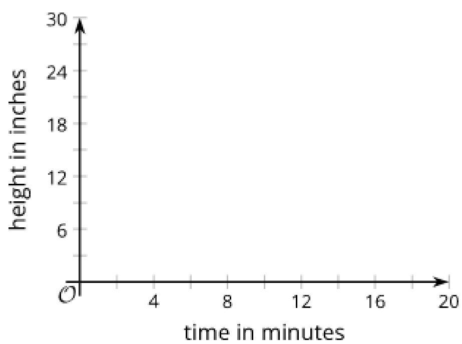
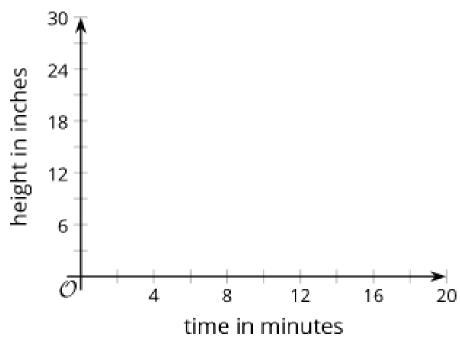
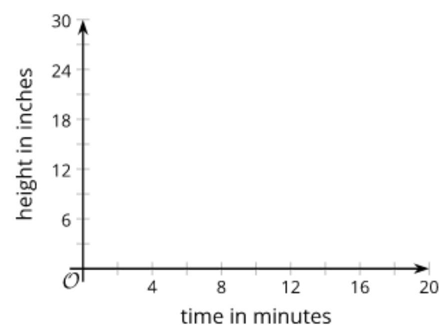


To prepare for a backyard party, a parent uses two identical hoses to fill a small pool that is 15 inches deep and a large pool that is 27 inches deep.

The height of the water in each pool is a function of time since the water is turned on.

Here are descriptions of three situations. For each situation, sketch the graphs of the two functions on the same coordinate plane, so that $S(t)$ is the height of the water in the small pool after t minutes, and $L(t)$ is the height of the water in the large pool after t minutes.

In both functions, the height of the water is measured in inches.

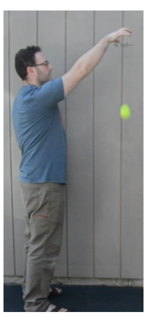
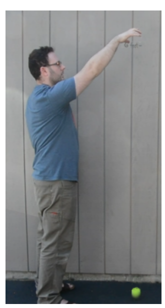
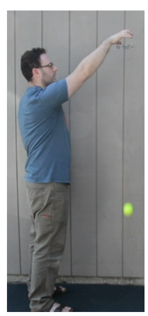
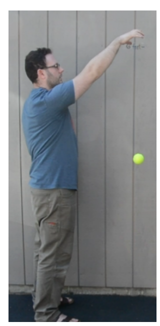
<table>
<tr>
<td>Situation 1: Each hose fills one pool at a constant rate. When the small pool is full, the water for that hose is shut off. The other hose keeps filling the larger pool until it is full.

</td>
<td>Situation 2: Each hose fills one pool at a constant rate. When the small pool is full, both hoses are shut off.

</td>
<td>Situation 3: Each hose fills one pool at a constant rate. When the small pool is full, both hoses are used to fill the large pool until it is full.

</td>
</tr>
</table>

Activity 4: The Bouncing Ball

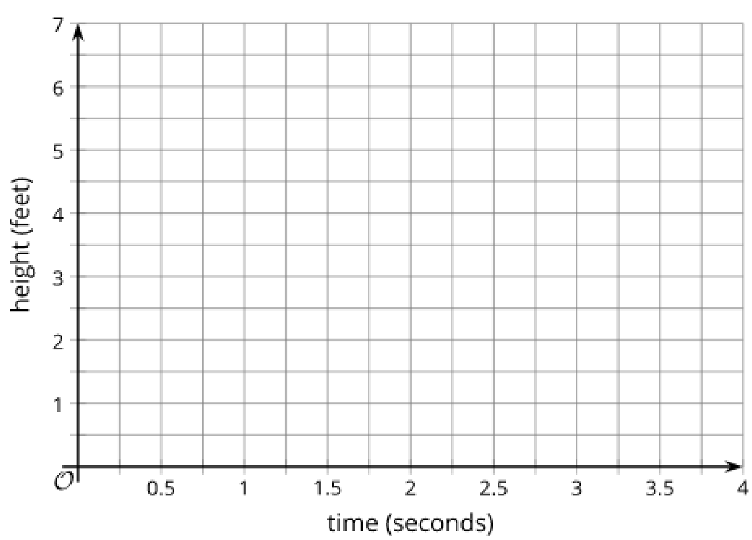
Here are some still images of a tennis ball being dropped. The height of the ball is a function of time. Suppose the height is h feet, t seconds after the ball is dropped.

1. To help you get started, here are some pictures and a table. Complete the table with your estimates before sketching your graph.

Time (seconds)	Height (feet)
0	
0.28	
0.54	
0.74	
1.03	
1.48	
1.88	
2.25	

0 seconds	0.28 seconds	0.54 seconds	0.74 seconds	1.03 seconds	1.48 seconds	1.88 seconds	2.25 seconds
							

2. Use the blank coordinate plane to sketch a graph of the height of the tennis ball as a function of time. Assume that the ball reaches a high point at 0, 1.03, and 1.88 seconds.



3. Identify horizontal and vertical intercepts of the graph. Explain what the coordinates tell us about the tennis ball.

4. Find the maximum and minimum values of the function. Explain what they tell us about the tennis ball.

Lesson Debrief

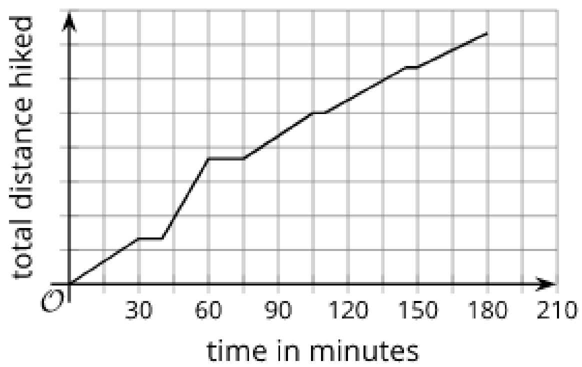
Lesson 8 Summary and Glossary

We can use graphs to help visualize the relationship between quantities in a situation, even if we only have a general description.

Here is a description of a hiker's journey on a trail:

A hiker walked briskly and steadily for about 30 minutes and then took a 10-minute break. Afterward, she jogged all the way to the end of the trail, which took about 20 minutes. There, she took a 15-minute break, and then started a leisurely walk back, stopping twice to enjoy the scenery. Her return trip along the same trail took 105 minutes.

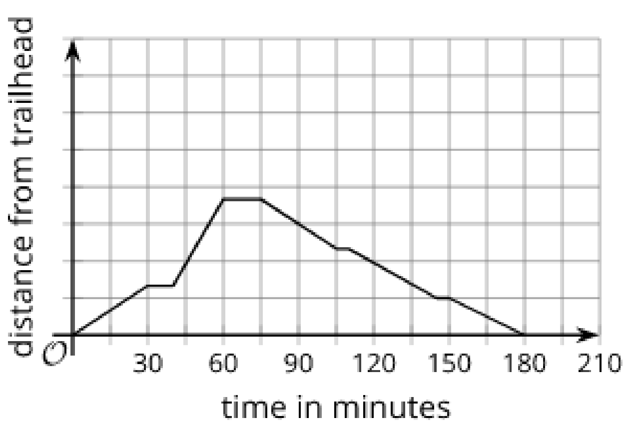
We can sketch a graph of the distance the hiker has traveled as a function of time based on this description.



Even though we don't know the specific distances she has traveled or the length of the trail, the graph can show some important features of the situation. For example:

- the intervals in which the distance increased or stayed constant
- when the distance was increasing more quickly or more slowly
- the time when the total distance hiked was greatest.

If we are looking at distance from the trailhead (the start of the trail) as a function of time, the graph of the function might look something like this:

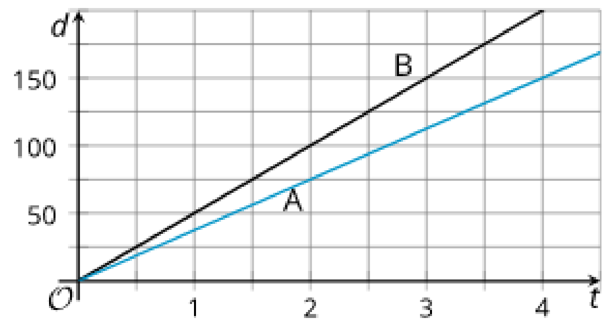


It shows the distance increasing as the hiker was walking away from the trailhead, and then decreasing as she was returning to the trailhead.

Unit 5 Lesson 8 Practice Problems

1. The graphs show the distance, d, traveled by two cars, A and B, over time, t. Distance is measured in miles and time is measured in hours.

   Which car traveled slower? Explain how you know.



2. Here are descriptions of four situations in which the volume of water in a tank is a function of time. Match each description to a corresponding graph.

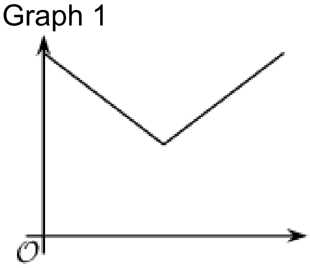
   a. An empty 20-gallon water tank is filled at a constant rate for 3 minutes until it is half full. Then, it is emptied at a constant rate for 3 minutes.

   b. A full 10-gallon water tank is drained for 30 seconds, until it is half full. Afterwards, it gets refilled.

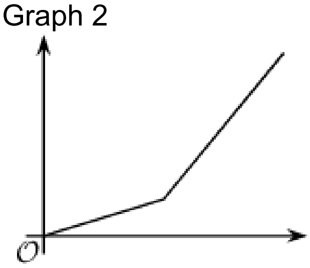
   c. A 2,000-gallon water tank starts out empty. It is being filled for 5 hours, slowly at first, and faster later.

   d. An empty 100-gallon water tank is filled in 50 minutes. Then, a dog jumps in and splashes around for 10 minutes, letting 7 gallons of water out. The tank is refilled afterwards.

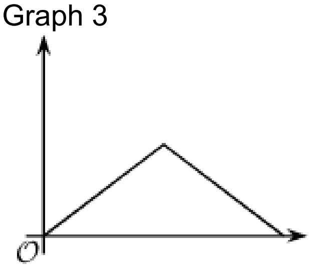
Graph 1



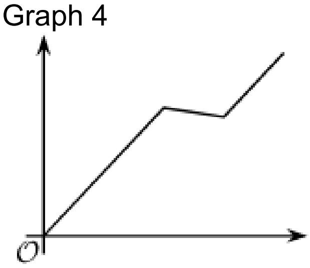
Graph 2



Graph 3



Graph 4



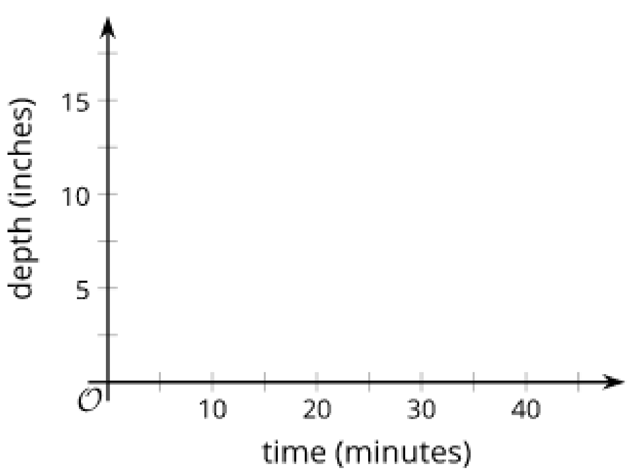
3. Clare describes her morning at school yesterday: "I entered the school on the first floor, then walked up to the third floor and stayed for my class for an hour. Afterwards, I had an hour-long class in the basement, and after that I went back to the ground level and sat outside to eat my lunch."

   Sketch a possible graph of her height from the ground floor as a function of time.

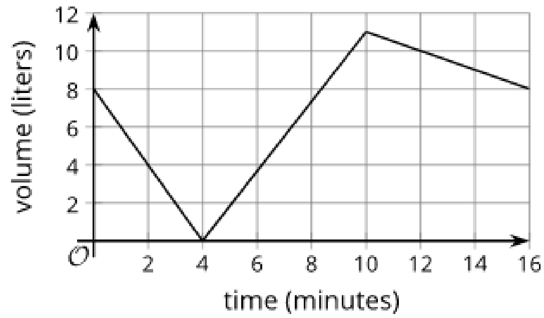
4. Tyler filled up their bathtub, took a bath, and then drained the tub. The function gives the depth of the water, in inches, t minutes after they began to fill the bathtub.

   These statements describe how the water level in the tub was changing over time. Use the statements to sketch an approximate graph of function.

   - $B(0) = 0$
   - $B(1) < B(7)$
   - $B(5) - B(0) = 6$
   - $B(9) = 11$
   - $B(10) = B(23)$
   - $B(20) > B(40)$



5. Function V gives the volume of water (liters) in a water cooler as a function of time, t (minutes). This graph represents function V.



   a. What is the greatest water volume in the cooler?

   b. Find the value or values of t that make $V(t) = 4$ true.
   Explain what the value or values tell us about the volume of the water in the cooler.

   c. Identify the horizontal intercept of the graph. What does it tell you about the situation?

(From Unit 5, Lesson 6)

6. Two functions are defined by these equations:

$$f(x) = 5.1 + 0.8x$$
$$g(x) = 3.4 + 1.2x$$

Which function has a greater value when x is 3.9? How much greater?

(From Unit 5, Lesson 5)

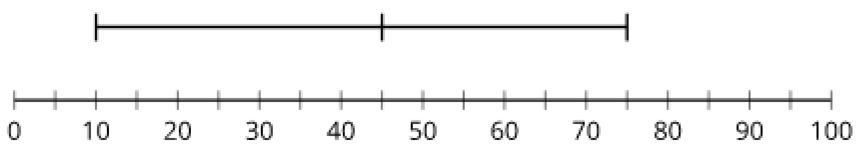
7. Function f is defined by the equation $f(x) = 3x - 7$. Find the value of c so that $f(c) = 20$ is true.

(From Unit 5, Lesson 5)

8. B is the midpoint of segment AC on the coordinate plane. The coordinates of the 3 points are: $A(-4,5);\ B(x,2)$; and $A(-4,5);\ B(x,2)$; and $C(9,y)$. What are the values of x and y?

(From Unit 3)

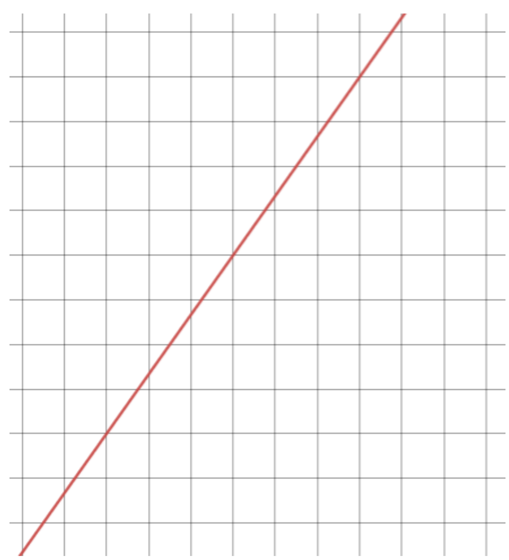
9. Noah draws this box plot for a data set that has an IQR of 0.



Explain why the box plot is complete even though there do not appear to be any boxes.

(From Unit 1)

10. For the line shown below:



a. What is the slope of the line?

b. Draw a line with a greater slope. What is the slope of your new line?

c. Draw a line with a smaller positive slope. What is the slope of your new line?

(Addressing NC.8.F.4)

Lessons 9 & 10: Checkpoint

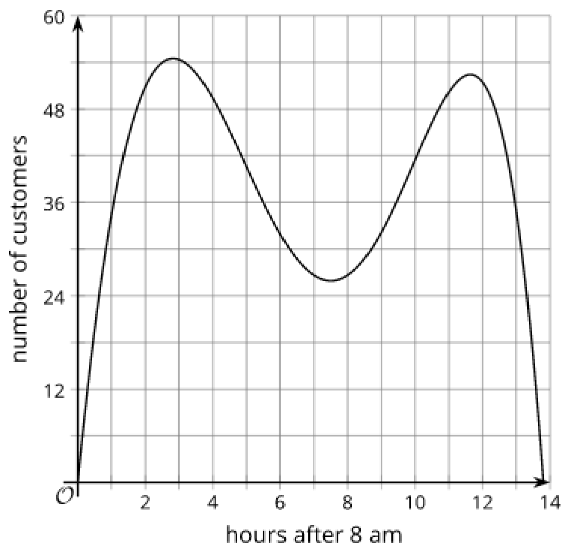
Learning Targets

- I can continue to grow as a mathematician and challenge myself.
- I can share what I know mathematically.

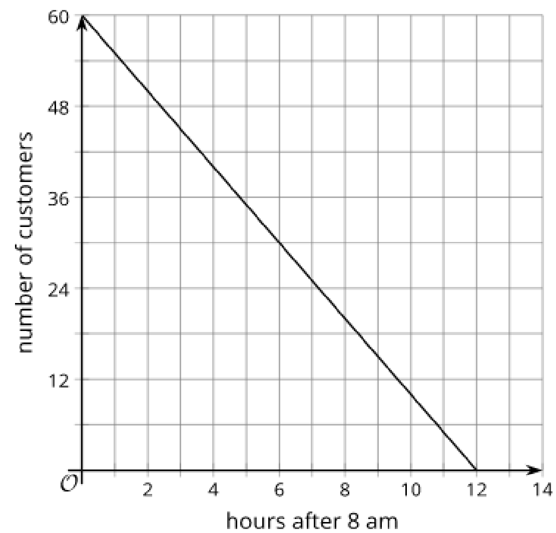
Station C: Graphs and Situations

1. These graphs show how busy restaurants are at different times of the day.

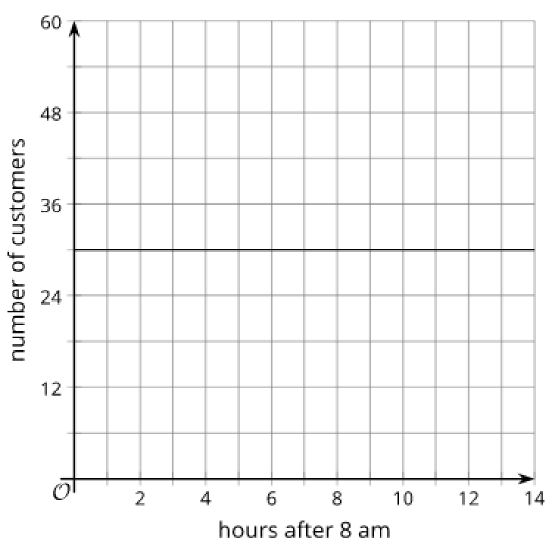
Restaurant A



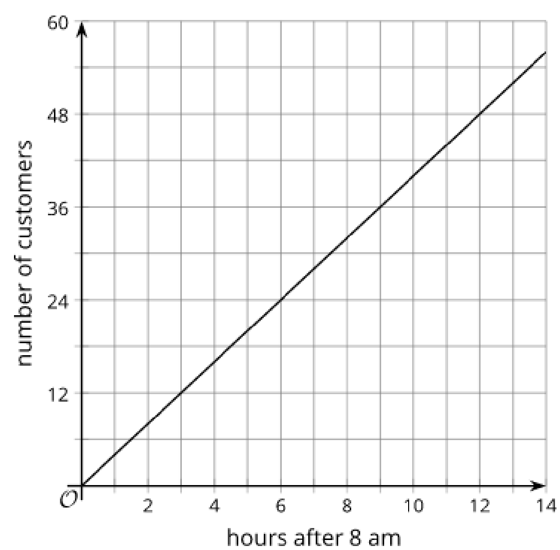
Restaurant B



Restaurant C



Restaurant D



a. Which restaurant is busy in the morning, then has fewer customers in the evening?

b. If Lin's mom wants to go to a popular dinner restaurant, which restaurant should Lin take her mom to eat?

c. Noah's dad prefers breakfast places with few customers so that he can start on work while eating. Which restaurant should Noah's dad go to for breakfast?

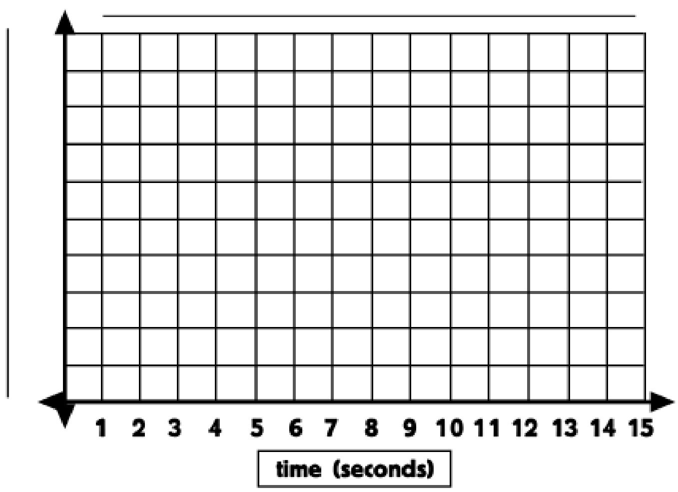
d. Which restaurant would you visit during a 30-minute lunch break?

2. Your teacher will give you a link to some videos. Choose any three videos to watch or watch the videos your teacher has assigned and follow the steps below:[1]

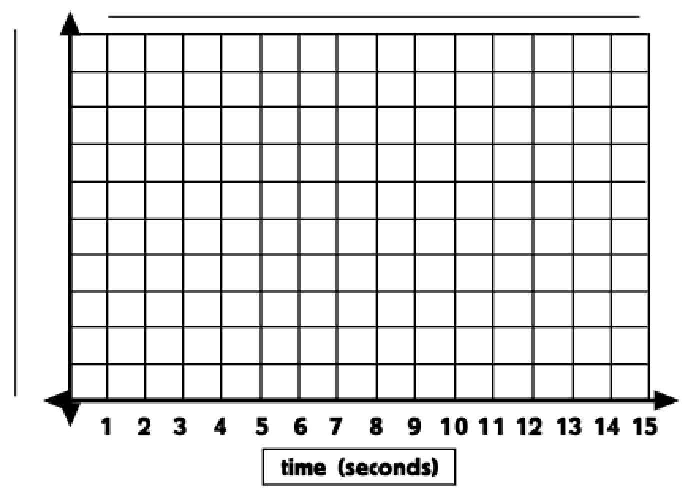
    a. Watch a video, pausing after you watch the slow-motion version.

    b. Notice the timer in the lower left of the video. Use this timer and your best estimate of the heights to graph the motion in the video. Pause or rewind the video as often as you need to.

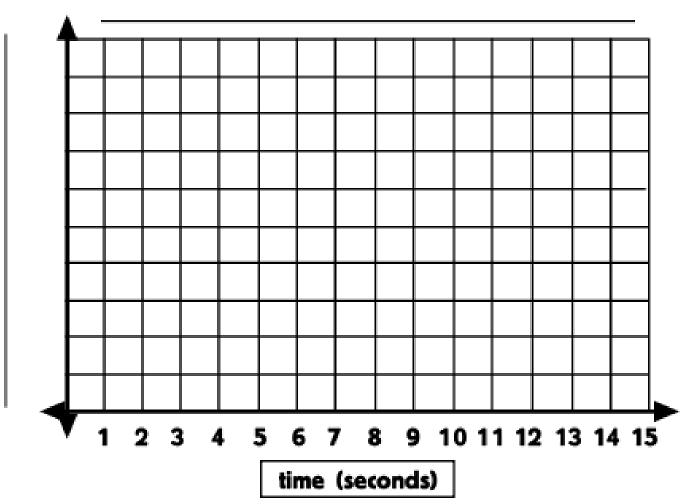
1.



2.



3.



    c. Watch the video to the end and compare your graph with the one in the video.

    d. Repeat as time permits.

[1] Adapted from www.graphingstories.com

Station D: Points into Function Notation and Back

1. A function is given by the equation $y = f(x)$. Write each of these coordinate pairs in function notation.

    a. $(2, 3)$

    b. $(-1, 4)$

    c. $(0, 3)$

    d. $(4, 0)$

    e. $(\frac{2}{3}, \frac{3}{4})$

2. A function is given by the equation $h(x) = 5x - 3$. Write the coordinate pair for the point associated with the given values in function notation.

    a. $h(3)$

    b. $h(-4)$

    c. $h(\frac{2}{5})$

3. Get ready to play this game: One person thinks of a rule for a function. The other person (or other people, working together) is the guesser. The guesser asks for the output of the function for certain inputs. ("What is $f(5)$?", for example.) After hearing the response to each question, the guesser can take a guess at the rule.

   Here are two sample games, played by Priya and Andre.

   Priya thinks of the rule $f(x) = 2x + 1$.

   Andre (the guesser): "What is $f(12)$?" (pronounced "f of 12")
   Priya: "25."
   Andre: "Is the rule $f(x) = x^2$?" (pronounced "f of x equals x squared")
   Priya: "No."

   Andre: "Okay, then - what is $f(3)$?"
   Priya: "7."
   Andre, thinks to himself: *The rule could be* $f(x) = 3x - 2$ *because* $3 \cdot 3 - 2 = 7$*. That doesn't work for* 12*, though, because* $3 \cdot 12 - 2 = 34$*, not* 25*. I need something that works for both. A simple way to make an output of* 7 *from an input of* 3 *is to multiply* 3 *by* 2*, then add* 1*. Does that work for an input of* 12*? Yes:* $2 \cdot 12 + 1 = 25$*.*
   Andre: "Is the rule $f(x) = 2x + 1$?"
   Priya: "Yes!"

   Since Andre guessed the rule, now they switch and Priya is the guesser.

   Andre thinks of the rule $f(x) = x^2 - 4$.

   Priya: "What is $f(4)$?"
   Andre: "12."
   Priya: "Is the rule $f(x) = 3x$?"
   Andre: "No."

   Priya: "What is $f(3)$?"
   Andre: "5."
   Priya, thinks to herself: *If the graph of this function is a line, it has a slope of* 7*.*
   Priya: "Is the rule $f(x) = 7x - 16$? That works for both of the guesses so far."
   Andre: "No."

   Priya: "What is $f(-4)$?"
   Andre: "12."
   Priya, thinks to herself: *Since my previous guess was wrong, I know this graph can't be a line. Maybe it involves* x^2*, since* $f(4)$ *is the same as* $f(-4)$*.*
   Priya: "Is the rule $f(x) = x^2 - 4$?"
   Andre: "Yes!"

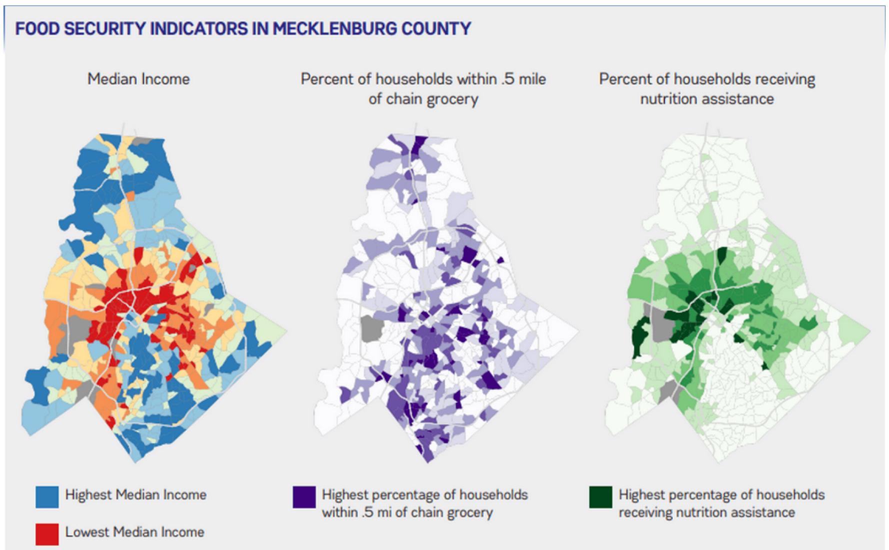
Now play the game with a partner or a group, with one person thinking of a rule and everyone else trying to guess it in as few turns as possible. The function rules can involve any mathematical operations you know. It may help the guesser(s) to plot the (input, output) coordinates on graph paper. After each round, switch so that someone else makes up the function rule.

Station E: Food Insecurity in Mecklenburg County

Recall being introduced to “food deserts” in Unit 4, Lesson 1. The data explored in that lesson was for San Antonio, Texas. Now you will explore data related to food insecurity in Mecklenburg County.

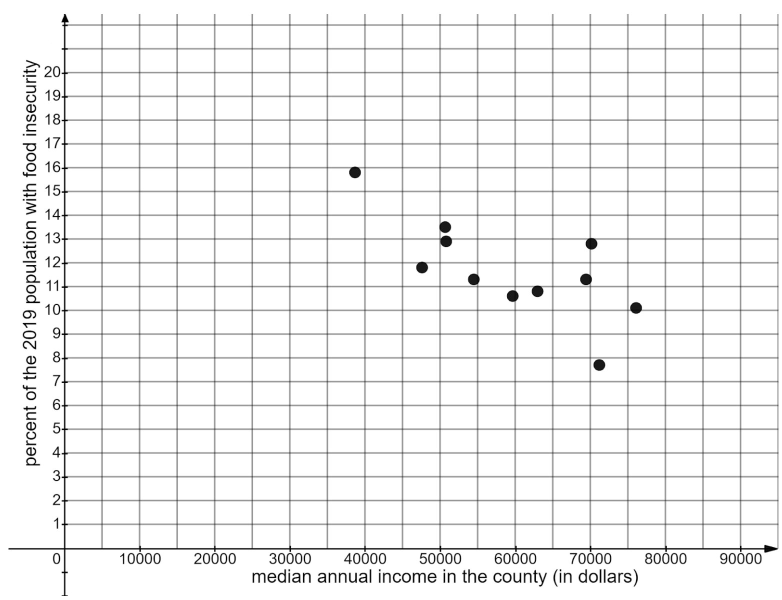
A household is considered to be food insecure when food intake of one or more household members is reduced and eating patterns are disrupted due to lack of money and resources. Food insecurity is defined as a lack of consistent access to enough food for every person in a household to live an active, healthy life.[2]

This graphic shows three factors that could be relevant to food insecurity. What do you notice? What do you wonder?



(Source[3])

One of the strategies Mecklenburg County has implemented to reduce food insecurity includes attempts to increase availability of fresh healthy food from farmers markets for families who qualify for government assistance for food. The data and scatter plot come from a study of counties around the country with varying levels of food insecurity in their populations.[4]



[2] Feeding America. *Food Insecurity.* https://www.feedingamerica.org/hunger-in-america/food-insecurity
[3] KarenKarp&Partners. (2017). *Unlocking the Potential of Charlotte’s Food Systems and Farmers’ Markets.* https://charlottenc.gov/HNS/CE/Documents/KKP_CharlotteFarmersMarketsFINAL.pdf
[4] Ibid.

Median annual income in the county	Percent of the 2019 population with food insecurity[5]
50685	13.5
71200	7.7
76097	10.1
70158	12.8
50815	12.9
59668	10.6
38681	15.8
54492	11.3
62978	10.8
69433	11.3
47607	11.8

The regression equation is given by the function $f(x) = -0.00013x + 19.36$, with x representing the median income and $f(x)$ representing the percentage of the population with food insecurity.

1. Interpret the slope and vertical intercept of the regression equation in terms of median income and percentage of the population with food insecurity.

2. The median annual income for Mecklenburg County when the data for this study was recorded was $62,978. Use the function that models this data, $f(x)$, to predict the percentage of food insecurity in 2019.

[5] Feeding America Action. *State-By-State Resource: The Impact of Coronavirus on Food Insecurity,* https://feedingamericaaction.org/resources/state-by-state-resource-the-impact-of-coronavirus-on-food-insecurity/

3. Compare the predicted value to the actual value, 10.8%. Is the percentage of the population with food insecurity in Mecklenburg County more or less than expected based upon the median annual income?

4. Do you think this comparison should be used as a means for celebrating or increasing concern regarding food insecurity in Mecklenburg County? Why or why not?

5. Do you believe that the farmers market program was effective in reducing food insecurity in Mecklenburg County? What other research could you do to find out?

Station F: Micro-Modeling[6]

Funds totaling $191,000 are designated for four schools. The distribution of the funds is to be in proportion to the number of students in each school. Student populations of the four schools are: School A, 386; School B, 1691; School C, 2109; School D, 817.

1. Figure out how much money each school gets.

2. Draw some sort of diagram to scale that helps show visually how the money is divided up among the four schools.

3. Find general formulas for deciding how much money each of the four schools gets in terms of the populations of the schools and the total amount of money to be distributed.

[6] Adapted from Achievethecore.org

Station G: Practice with Rational Bases

Some mathematicians expand expressions involving exponents when they get stuck when evaluating an expression. The expansion of $2^5 \cdot 2^3$ is $(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2)$. How might expanding help you finish evaluating the expression?[7]

1. Use expansion to evaluate the following expressions involving exponents.

   a. $7^5 \cdot 7^6$ b. $(\frac{2}{3})^3$ c. $\frac{3^5}{3^2}$

Some mathematicians use these symbolic equations to define exponents rules. See exponent rules A-E below:

A. $x^n \cdot x^m = x^{n+m}$	B. $(x^n)^m = x^{n \cdot m}$	C. $\frac{x^n}{x^m} = x^{n-m}$	D. $x^{-n} = \frac{1}{x^n}$	E. $x^0 = 1$

2. Match each of the expressions to a rule or rules that could be used to help evaluate it and then evaluate the expression. The first row has been completed for you as an example.

Expression	Letter corresponding to rule or rules above	Evaluate	Expression	Letter corresponding to rule or rules above	Evaluate
$2^8 \cdot 2^{-8}$	Rules A and E	$2^8 \cdot 2^{-8} = 2^{(8+-8)} = 2^0 = 1$	11^{-8}		
$(7^2)^3$			$(6^{-3})^5$		
$\frac{6^5}{6^{-8}}$			$(\frac{5}{6})^4(\frac{5}{6})^5$		
$(3^4)^0$			$\frac{10^5}{10^5}$		

[7] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

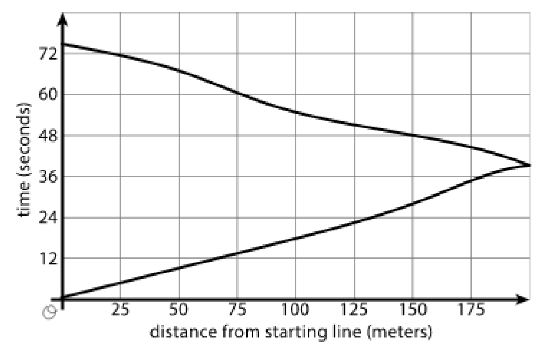
Lesson 11: Comparing Graphs

Learning Targets

- I can compare the features of graphs of functions and explain what they mean in the situations represented.
- I can make sense of an equation of the form $f(x) = g(x)$ in terms of a situation and a graph and know how to find the solutions.
- I can make sense of statements about two or more functions when they are written in function notation.

Bridge

Priya is running once around the track. The graph shows her time given how far she is from her starting point.[1]



1. What was her farthest distance from her starting point?

2. Estimate how long it took her to run around the track.

3. Estimate when she was 100 meters from her starting point. How do you know?

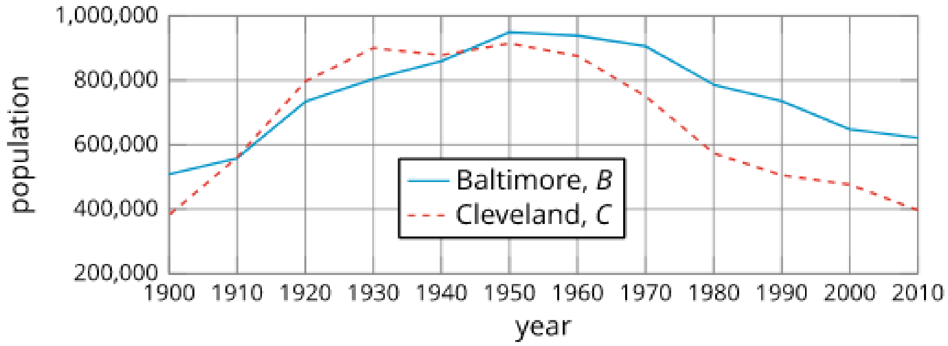
4. Estimate how far she was from the starting line after 60 seconds. How do you know?

5. What does the point $(150, 48)$ represent?

[1] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

Warm-up: Population Growth

This graph shows the populations of Baltimore and Cleveland in the 20th century. $B(t)$ is the population of Baltimore in year t. $C(t)$ is the population of Cleveland in year t.



1. Estimate $B(1930)$ and explain what it means in this situation.

2. Here are pairs of statements about the two populations. In each pair, which statement is true? Be prepared to explain how you know.

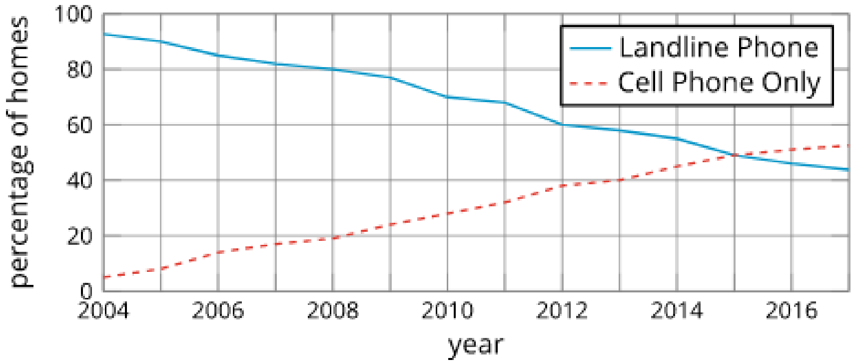
   a. $B(2000) > C(2000)$ or $B(2000) < C(2000)$

   b. $B(1900) = C(1900)$ or $B(1900) > C(1900)$

3. Were the two cities' populations ever the same? If so, when?

Activity 1: Wired or Wireless?

$H(t)$ is the percentage of homes in the United States that have a landline phone in year t. $C(t)$ is the percentage of homes with *only* a cell phone. Here are the graphs of H and C.



1. Estimate $H(2006)$ and $C(2006)$. Explain what each value tells us about the phones.

2. What is the approximate value of t when $C(t) = 20$? Explain what that value of t means in this situation.

3. Determine if each equation is true. Be prepared to explain how you know.

   a. $C(2011) = H(2011)$

   b. $C(2015) = H(2015)$

4. Interpret the following statement: “For $t > 2015$, $C(t) > H(t)$.” Is the statement true?

5. Between 2004 and 2015, did the percentage of homes with landlines decrease at the same rate at which the percentage of cell-phones-only homes increased? Explain or show your reasoning.

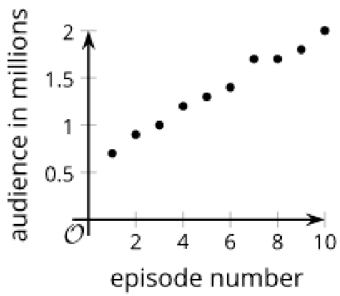
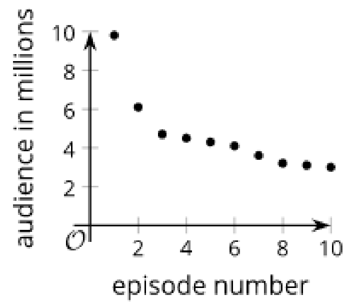
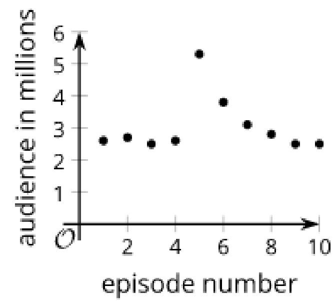
Are You Ready For More?

1. Explain why the statement $C(t) + H(t) \leq 100$ is true in this situation.

2. What value does $C(t) + H(t)$ appear to take between 2004 and 2017? How much does this value vary in that interval?

Activity 2: Audience of TV Shows

The number of people who watched a TV episode is a function of that show's episode number. Here are three graphs of three functions—A, B, and C—representing three different TV shows.

Show A	Show B	Show C
 Description: __________	 Description: __________	 Description: __________

1. Match each description with a graph that could represent the situation described. One of the descriptions has no corresponding graph.

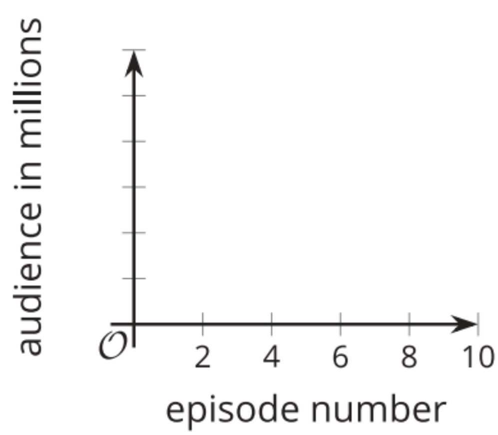
   a. This show has a good core audience. They had a guest star in the fifth episode that brought in some new viewers, but most of them stopped watching after that.

   b. This show is one of the most popular shows, and its audience keeps increasing.

   c. This show has a small audience, but it's improving, so more people are noticing.

   d. This show started out huge. Even though it feels like it crashed, it still has more viewers than the other two shows.

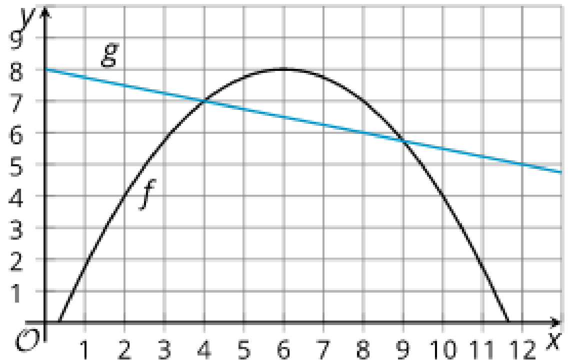
2. Which is greatest, $A(7)$, $B(7)$, or $C(7)$? Explain what the answer tells us about the shows.



3. Sketch a graph of the viewership of the fourth TV show that did not have a matching graph.

Activity 3: Functions f and g

1. Here are graphs that represent two functions, f and g.



   Decide which function value is greater for each given input. Be prepared to explain your reasoning.

   a. $f(2)$ or $g(2)$

   b. $f(4)$ or $g(4)$

   c. $f(6)$ or $g(6)$

   d. $f(8)$ or $g(8)$

2. Is there a value of x for which the equation $f(x) = g(x)$ is true? Explain your reasoning.

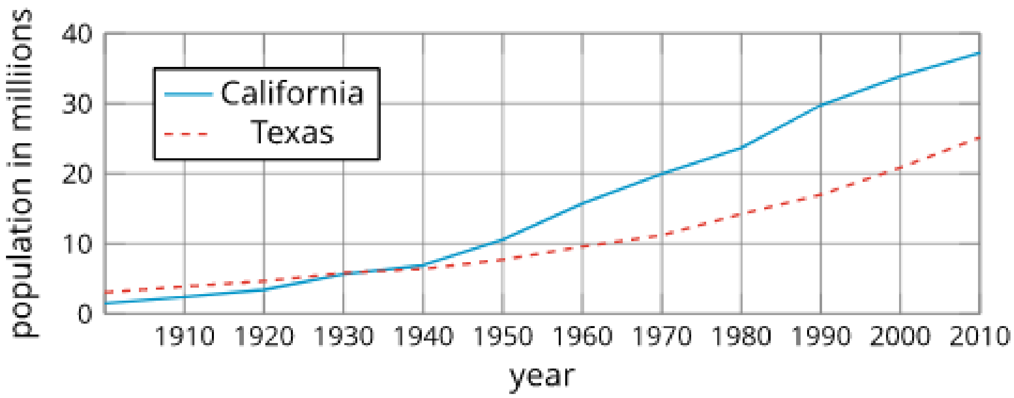
3. Identify at least two values of x for which the inequality $f(x) < g(x)$ is true.

Lesson Debrief

What can we tell about the populations?	How can we tell?	How can we convey this with function notation?
In 2010, Baltimore had more people than Cleveland.		
Baltimore and Cleveland had the same population twice in the past century, in 1910 and around 1944.		
After the mid-1940s, Cleveland has had a smaller population than Baltimore.		
In the first half of the 20th century, the population of Cleveland grew at a faster rate than that of Baltimore.		
Since 1950, the population of Cleveland has dropped at a faster rate than that of Baltimore.		

Lesson 11 Summary and Glossary

Graphs are very useful for comparing two or more functions. Here are graphs of functions C and T, which give the populations (in millions) of California and Texas in year x.

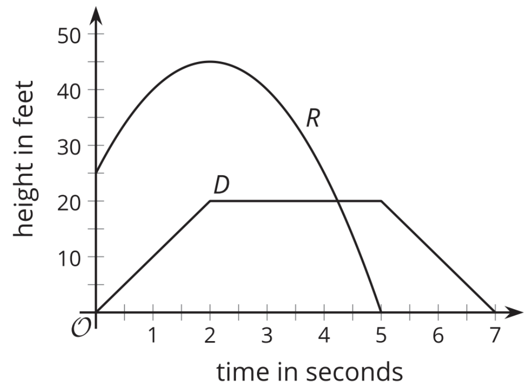


What can we tell about the populations?	How can we tell?	How can we convey this with function notation?
In the early 1900s, California had a smaller population than Texas.	The graph of C is below the graph of T when x is 1900.	$C(1900) < T(1900)$
Around 1935, the two states had the same population of about 5 million people.	The graphs intersect at about $(1935, 5)$.	$C(1935) = 5$ and $T(1935) = 5$, and $C(1935) = T(1935)$
After 1935, California has had more people than Texas.	When x is greater than 1935, the graph of $C(x)$ is above that of $T(x)$.	$C(x) > T(x)$ for $x > 1935$
Both populations have increased over time, with no periods of decline.	Both graphs slant upward from left to right.	
From 1900 to 2010, the population of California has risen faster than that of Texas. California had a greater average rate of change.	If we draw a line to connect the points for 1900 and 2010 on each graph, the line for C has a greater slope than that for T.	

Unit 5 Lesson 11 Practice Problems

1. Functions R and D give the height, in feet, of a toy rocket and a drone, t seconds after they are released.

   Here are the graphs of R (for the rocket) and D (for the drone).

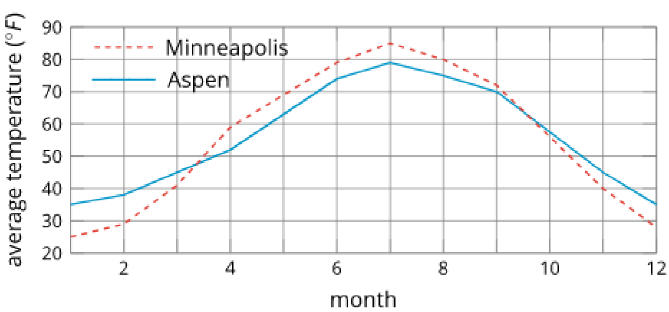


   Write each statement in function notation:

   a. At 3 seconds, the toy rocket is higher than the drone.

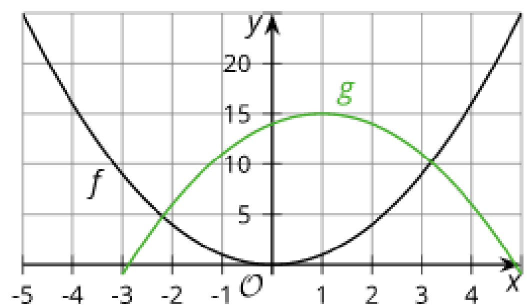
   b. At the start, the toy rocket is 25 feet above the drone.

2. $A(t)$ is the average high temperature in Aspen, Colorado, t months after the start of the year. $M(t)$ is the average high temperature in Minneapolis, Minnesota, t months after the start of the year. Temperature is measured in degrees Fahrenheit.



   Which function had the higher average rate of change between the beginning of January ($t = 1$) and middle of March $(t \approx 3.5)$? What does this mean about the temperature in the two cities?

3. Here are two graphs representing functions f and g.



Select **all** statements that are true about functions f and g.

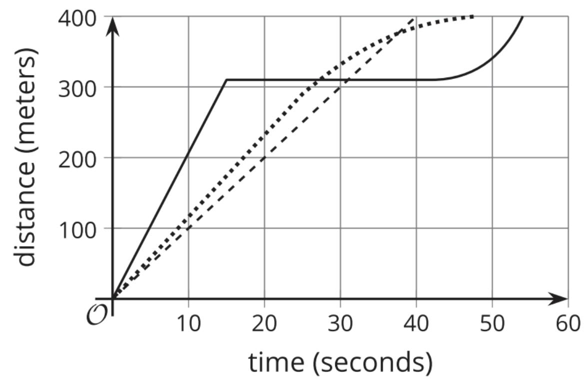
a. $f(0) > g(0)$

b. There are two values of x where $f(x) = g(x)$.

c. If $g(x) = 15$, then $x = 1$.

d. $f(-3) > g(4)$

4. The three graphs represent the progress of three runners in a 400-meter race.



The solid line represents runner A. The dotted line represents runner B. The dashed line represents runner C.

a. One runner ran at a constant rate throughout the race. Which one?

b. A second runner stopped running for a while. Which one? During which interval of time did that happen?

c. Describe the third runner's race. Be as specific as possible.

d. Who won the race? Explain how you know.

5. Function f is represented by $f(x) = 5(x + 11)$.

    a. Find $f(-2)$.

    b. Find the value of x such that $f(x) = 90$ is true.

(From Unit 5, Lesson 5)

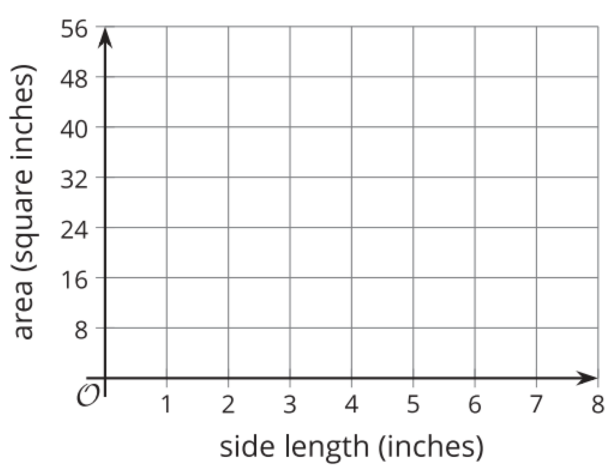
6. Function A gives the area, in square inches, of a square with side length x inches.

    a. Complete the table.

x	0	1	2	3	4	5	6
$A(x)$							

    b. Represent function A using an equation.

    c. Sketch a graph of function A.



(From Unit 5, Lesson 4)

7. Solve the following system of equations. In which quadrant is the point that represents the solution on the coordinate plane? Explain or show your reasoning.

$$\begin{cases} y = 3x + 6 \\ x = 2y - 7 \end{cases}$$

(From Unit 3)

8. On a math test, there are multiple choice questions worth 4 points and open-ended questions worth 6 points. Mr. Pills' first period class won a competition, and each student had 10 extra credit points added to their test score!

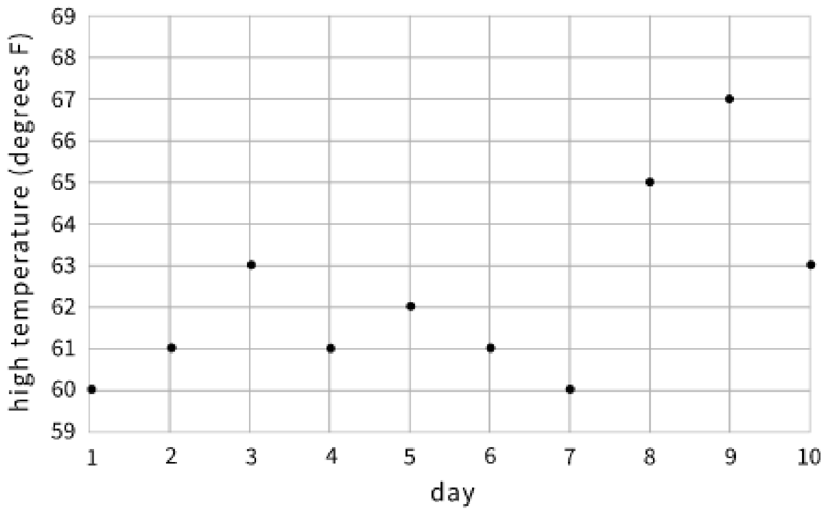
    a. If Mai answered 13 multiple choice questions correctly and earned a total score of 92, how many open-ended questions did she answer correctly?

    b. If Priya answered 4 open-ended questions correctly and earned a total score of 94, how many multiple choice questions did she answer correctly?

    c. Write an expression that will represent a students' score in first period who answers *m* multiple choice questions and *e* open-ended questions correctly.

(From Unit 2)

9. The graph below shows the high temperatures in a city over a 10-day period.[2]



a. What was the highest temperature in the city during the 10-day period?

b. What was the temperature on Day 3?

c. On what day was the temperature 65°?

d. What does the point (5, 62) represent on the graph?

(Addressing NC.8.F.1)

[2] Adapted from IM 6–8 Math https://curriculum.illustrativemathematics.org/MS/index.html, which was originally developed by Open Up Resources and authored by Illustrative Mathematics, and is copyright 2017–2019 by Open Up Resources. It is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0). OUR's 6–8 Math Curriculum is available at https://openupresources.org/math-curriculum/. Adaptations and updates to IM 6–8 Math are copyright 2019 by Illustrative Mathematics, and are licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0).

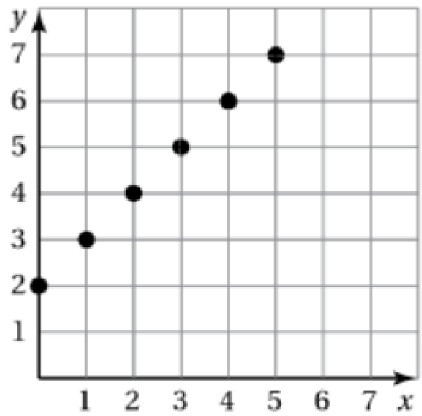
Lesson 12: Domain and Range (Part One)

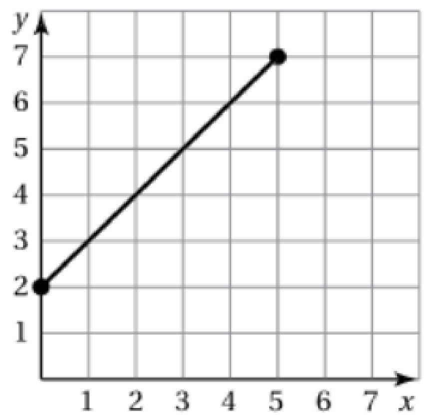
Learning Targets

- When given a description of a function in a situation, I can determine a reasonable domain and range for the function.
- I know what is meant by the “domain” and “range” of a function.

Bridge

Observe the two graphs. What do you notice? What do you wonder?





Warm-up: Number of Barks

Earlier, you saw a situation where the total number of times a dog has barked was a function of the time, **in seconds**, after its owner tied its leash to a post and left to go into a store. Less than 3 minutes after he left, the owner returned, untied the leash, and walked away with the dog.

1. Could each value be an input of the function? Be prepared to explain your reasoning.

   15 $\quad$ $84\frac{1}{2}$ $\quad$ 300

2. Could each value be an output of the function? Be prepared to explain your reasoning.

   15 $\quad$ $84\frac{1}{2}$ $\quad$ 300

Activity 1: Possible or Impossible?

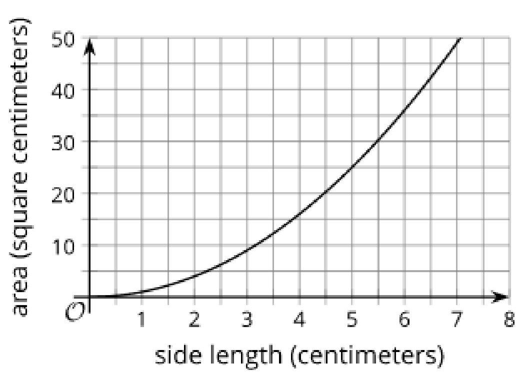
Decide whether each of the following numbers is a possible input for the functions described here. Sort the numbers into two groups—possible inputs and impossible inputs. Record your sorting decisions in the table.

Functions	**Circle the <u>possible inputs</u> and cross out the <u>impossible inputs</u>**
1. The area of a square, in square centimeters, is a function of its side length, s, in centimeters. The equation $A(s) = s^2$ defines this function.	-3 9 $\frac{3}{5}$ 15 0.8 4 0 $\frac{25}{4}$ 0.001 -18 6.8 72
2. A tennis camp charges \$40 per student for a full-day camp. The camp runs only if at least 5 students sign up, and it limits the enrollment to 16 campers a day. The amount of revenue, in dollars, that the tennis camp collects is a function of the number of students that enroll. The equation $R(n) = 40n$ defines this function.	-3 9 $\frac{3}{5}$ 15 0.8 4 0 $\frac{25}{4}$ 0.001 -18 6.8 72
3. The relationship between temperature in Celsius and the temperature in Kelvin can be represented by a function k. The equation $k(c) = c + 273.15$ defines this function, where c is the temperature in Celsius and $k(c)$ is the temperature in Kelvin.	-3 9 $\frac{3}{5}$ 15 0.8 4 0 $\frac{25}{4}$ 0.001 -18 6.8 72

Activity 2: What About the Outputs?

In an earlier activity, you saw a function representing the area of a square (function A) and another representing the revenue of a tennis camp (function R). Refer to the descriptions of those functions to answer these questions.

1. Here is a graph that represents function A, defined by $A(s) = s^2$, where s is the side length of the square in centimeters.



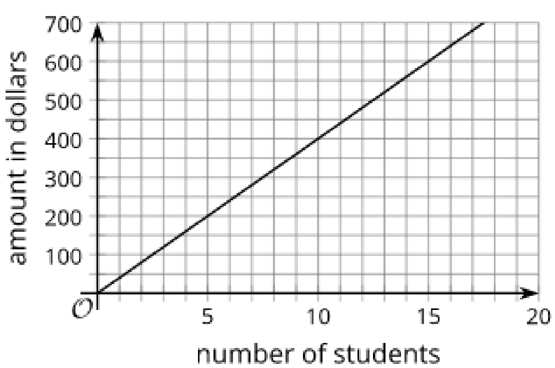
   a. Name three possible input-output pairs of this function.

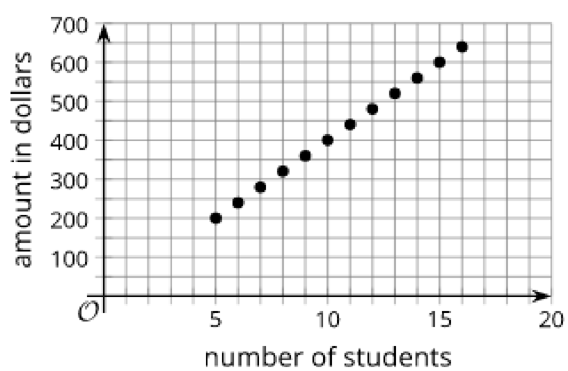
b. Earlier we described the set of all possible input values of A as "any number greater than or equal to 0." How would you describe the set of all possible output values of A?

2. Function R is defined by $R(n) = 40n$, where n is the number of campers.

a. Is 20 a possible output value in this situation? What about 100? Explain your reasoning.

b. Here are two graphs that relate the number of students and camp revenue in dollars. Which graph could represent function R? Explain why the other one could not represent the function.





c. Describe the set of all possible output values of R.

Are You Ready For More?

If the camp wishes to collect at least $500 from the participants, how many students can they have? Explain how this information is shown on the graph.

Lesson Debrief

	In the domain?	**In the range?**
Negative values		
0		
Values less than 1		
24		
25		
60		
Fractions		
Values greater than 480		
1,500		

Lesson 12 Summary and Glossary

The **domain** of a function is the set of all possible input values. Depending on the situation represented, a function may take all numbers as its input or only a limited set of numbers.

Domain: The domain of a function is the set of all of its possible input values.

- Function A gives the area of a square, in square centimeters, as a function of its side length, s, in centimeters.
  - The input of A can be 0 or any positive number, such as 4, 7.5, or $\frac{19}{3}$. It cannot include negative numbers because lengths cannot be negative. This means the domain of A includes 0 and all positive numbers ($s \geq 0$).
- Function q gives the number of buses needed for a school field trip as a function of the number of people, n, going on the trip.
  - The input of q can be 0 or positive whole numbers because a negative or fractional number of people doesn't make sense. If the number of people at a school is 120, then the domain is limited to all non-negative whole numbers up to 120 ($0 \leq n \leq 120$).
- Function v gives the total number of visitors to a theme park as a function of days, d, since a new attraction was open to the public.
  - The input of v can be positive or negative. A positive input means days since the attraction was open, and a negative input means days before the attraction was open.
  - The input can also be whole numbers or fractional. The statement $v(17.5)$ means 17.5 days after the attraction was open.
  - This means that the domain of v includes all numbers. If the theme park had been opened for exactly one year before the new attraction was open, then the domain would be all numbers greater than -365 (or $d \geq -365$).

The **range** of a function is the set of all possible output values. Once we know the domain of a function, we can determine the range that makes sense in the situation.

Range: The range of a function is the set of all of its possible output values.

- The output of function A is the area of a square in square centimeters, which cannot be negative but can be 0 or greater, not limited to whole numbers. The range of A is 0 and all positive numbers.
- The output of q is the number of buses, which can only be 0 or positive whole numbers. If there are 120 people at the school, however, and if each bus could seat 30 people, then only up to 4 buses are needed. The range that makes sense in this situation would be any whole number that is at least 0 and at most 4.
- The output of function v is the number of visitors, which cannot be fractional or negative. The range of v therefore includes 0 and all positive whole numbers.

Unit 5 Lesson 12 Practice Problems

1. The cost for an upcoming field trip is $30 per student. The cost of the field trip C, in dollars, is a function of the number of students x.

   Select **all** the possible outputs for the function defined by $C(x) = 30x$.

   a. 20
   b. 30
   c. 50
   d. 90
   e. 100

2. A rectangle has an area of 24 cm^2. Function f gives the length of the rectangle, in centimeters, when the width is w cm.

   Determine if each value, in centimeters, is a possible input of the function.

   3 0.5 48 -6 0

3. Select **all** the possible input-output pairs for the function $y = -3x^2 - 4$.

   a. $(-1, -1)$
   b. $(-2, 8)$
   c. $(1, -7)$
   d. $(0, -4)$
   e. $(3, -23)$
   f. $(4, -52)$

   Is 0 in the range of this function? How do you know?

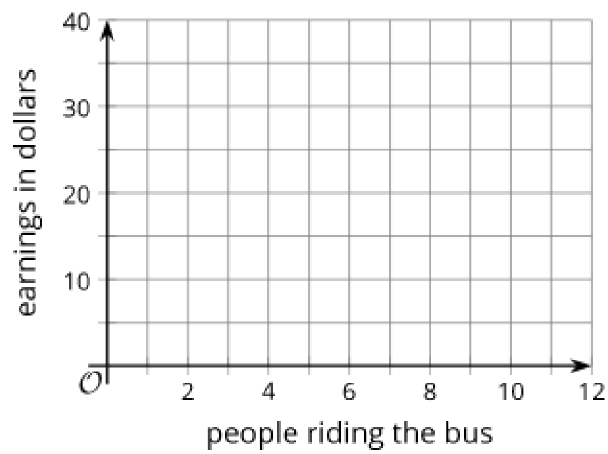
   Describe the range of this function.

4. A small bus charges $3.50 per person for a ride from the train station to a concert. The bus will run if at least three people take it, and it cannot fit more than 10 people.

   Function B gives the amount of money that the bus operator earns when n people ride the bus.

   a. Identify all numbers that make sense as inputs and outputs for this function.

   b. Sketch a graph of B.



5. To raise funds for a trip, members of a high school math club are holding a game night in the gym. They sell tickets at $5 per person. The gym holds a maximum of 250 people. The amount of money raised is a function of the number of tickets sold.

   Which statement accurately describes the domain of the function?

   a. all numbers less than 250

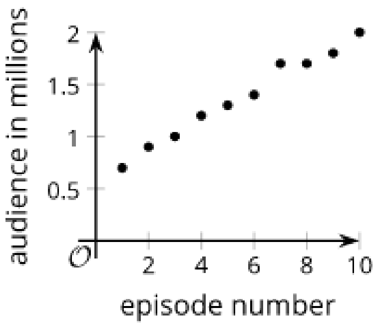
   b. all integers

   c. all positive integers

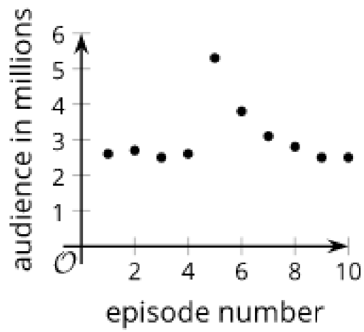
   d. all positive integers less than or equal to 250

6. The graphs show the audience, in millions, of two TV shows as a function of the episode number.

Show A



Show C



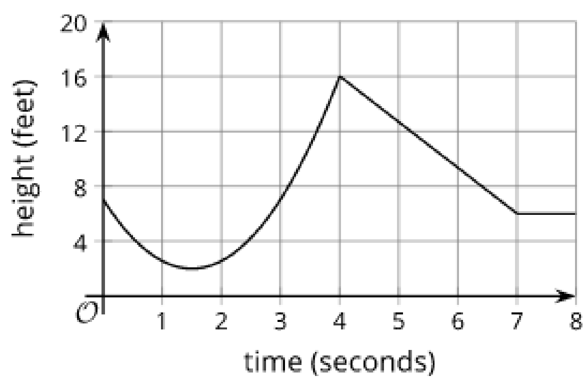
For each show, try to pick two episode numbers between which the function has a negative average rate of change. Either estimate this average rate of change, or explain why it is not possible.

(From Unit 5, Lesson 11)

7. Match each feature of the graph with the corresponding coordinate point.

If the feature does not exist, choose "none."

a.	maximum	i.	$(0, 7)$
b.	minimum	ii.	$(1.5, 2)$
c.	vertical intercept	iii.	$(4, 16)$
d.	horizontal intercept	iv.	none



(From Unit 5, Lesson 6)

8. Two functions are defined by the equations $f(x) = 5 - 0.2x$ and $g(x) = 0.2(x + 5)$.

   Select **all** statements that are true about the functions.

   a. $f(3) > 0$

   b. $f(3) > 5$

   c. $g(-1) = 0.8$

   d. $g(-1) > 0.5$

   e. $f(0) = g(0)$

   (From Unit 5, Lesson 5)

9. The graph of function f passes through the coordinate points $(0, 3)$ and $(4, 6)$.

   Use function notation to write the information each point gives us about function f.

   (From Unit 5, Lesson 3)

10. Here is a system of equations: $\begin{cases} 7x - 4y = -11 \\ 7x + 4y = -59 \end{cases}$

    Would you rather use subtraction or addition to solve the system? Explain your reasoning.

    (From Unit 3)

11. The videography team entered a contest and won a monetary prize of $1,350. Which expression represents how much each person would get if there were x people on the team?

    a. $\frac{1350}{x}$

    b. $1350 + x$

    c. $\frac{1350}{5}$

    d. $1350 - x$

    (From Unit 2)

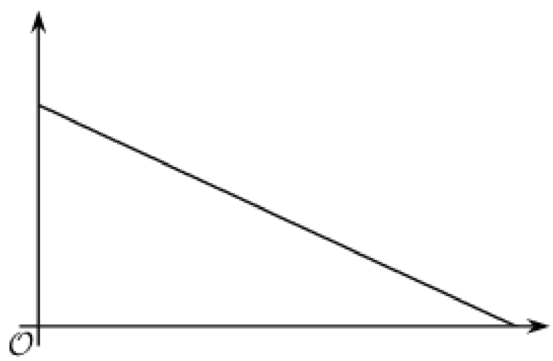
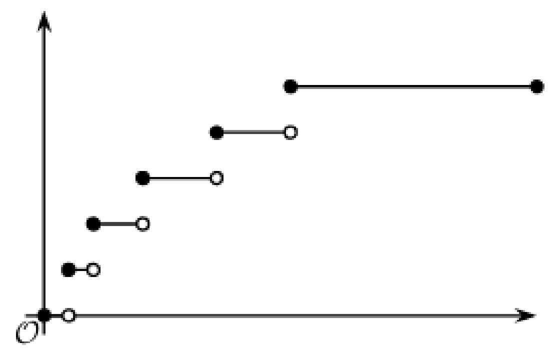
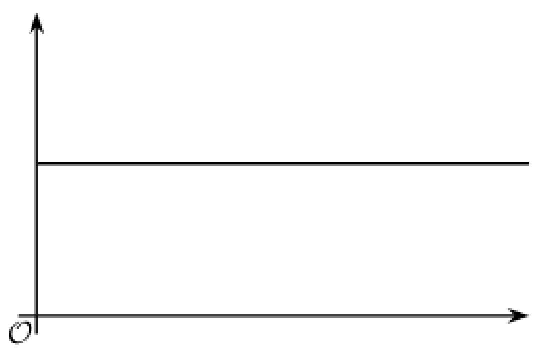
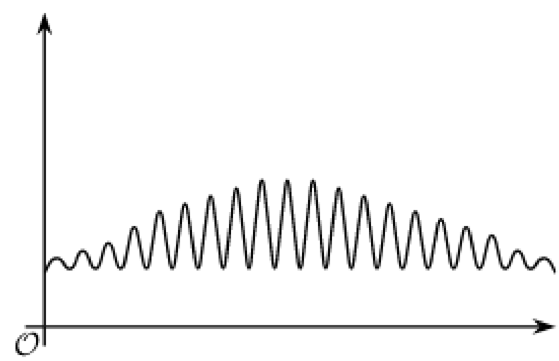
Lesson 13: Domain and Range (Part Two)

Learning Target

- When given a description of a function in a situation, I can determine a reasonable domain and range for the function.

Warm-up: Unlabeled Graphs

Which one doesn't belong? Explain your reasoning.

a. 	b. 
c. 	d. 

Activity 1: Time on the Swing

A child gets on a swing in a playground, swings for 30 seconds, and then gets off the swing.

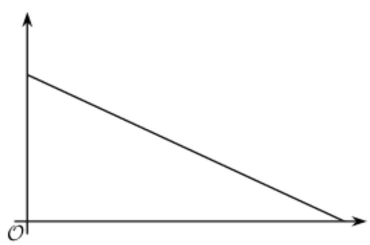
1. Here are descriptions of four functions relating to the situation and four graphs representing them.

   The independent variable in each function is time, measured in seconds.

   Match each function with a graph that could represent it. Then, label the axes with the appropriate variables. Be prepared to explain how you made your matches.

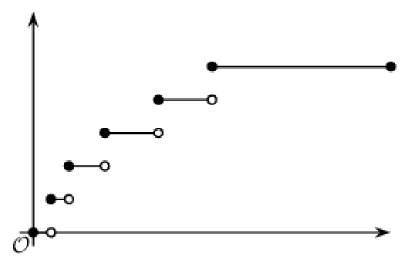
- Function h: The height of the swing, in feet, as a function of time since the child gets on the swing
- Function r: The amount of time left on the swing as a function of time since the child gets on the swing
- Function d: The distance, in feet, of the swing from the top beam (from which the swing is suspended) as a function of time since the child gets on the swing
- Function s: The total number of times an adult pushes the swing as a function of time since the child gets on the swing

Graph A Function: _____



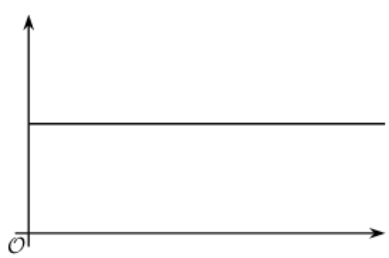
Domain: Range:

Graph B Function: _____



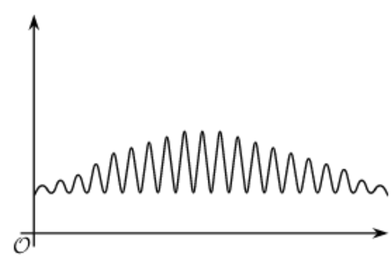
Domain: Range:

Graph C Function: _____



Domain: Range:

Graph D Function: _____



Domain: Range:

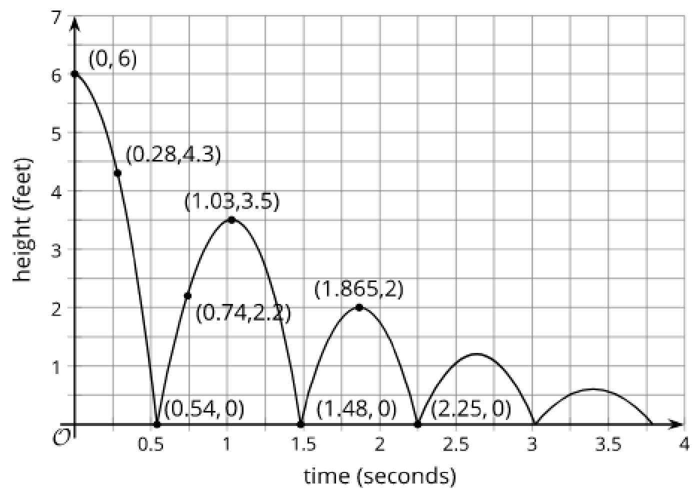
2. On each graph, mark one or two points that—if you had the coordinates—could help you determine the domain and range of the function. Be prepared to explain why you chose those points.

3. Once you receive the information you need from your teacher, describe the domain and range that would be reasonable for each function in this situation.

Activity 2: Back to the Bouncing Ball

A tennis ball was dropped from a certain height. It bounced several times, rolled along for a short period, and then stopped. Function H gives its height over time.

Here is a partial graph of H. Height is measured in feet. Time is measured in seconds.



Use the graph to help you answer the questions.

Be prepared to explain what each value or set of values means in this situation.

1. Find $H(0)$.

2. Find the value of x when $H(x) = 0$.

3. Describe the domain of the function.

4. Describe the range of the function.

Are You Ready For More?

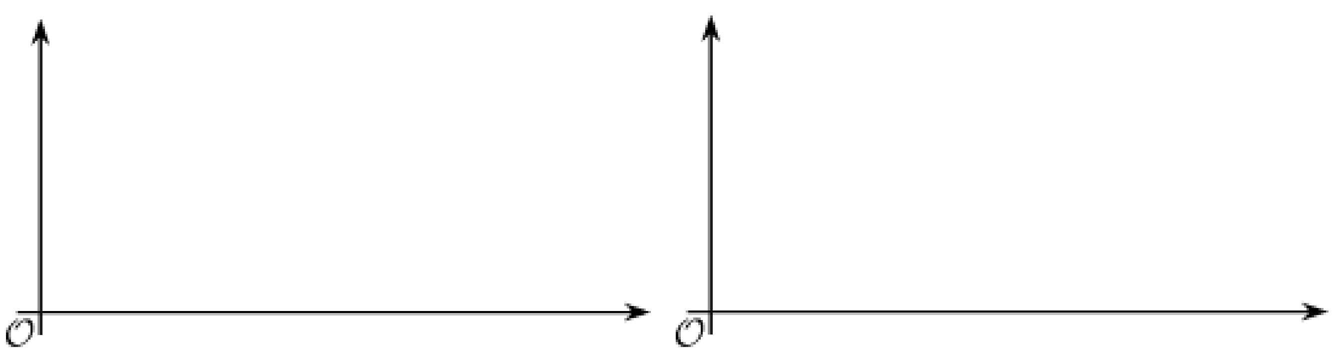
In function H, the input was time in seconds and the output was height in feet.

Think about some other quantities that could be inputs or outputs in this situation.

1. Describe a function whose domain includes only integers. Be sure to specify the units.

2. Describe a function whose range includes only integers. Be sure to specify the units.

3. Sketch a graph of each function.



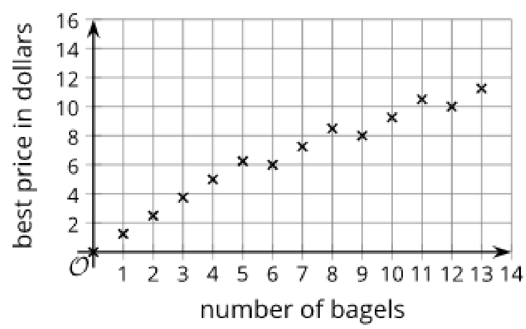
Lesson Debrief

Lesson 13 Summary and Glossary

The graph of a function can sometimes give us information about its domain and range.

Here are graphs of two functions we saw earlier in the unit. The first graph represents the best price of bagels as a function of the number of bagels bought. The second graph represents the height of a bungee jumper as a function of seconds since the jump began.

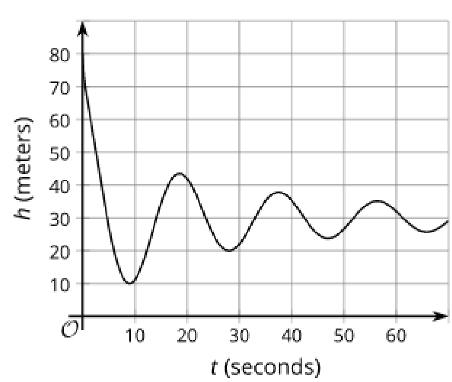
What are the domain and range of each function?



The number of bagels cannot be negative but could include 0 (no bagels bought). The domain of the function therefore includes 0 and positive whole numbers, or $n \geq 0$.

The best price can be \$0 (for buying 0 bagels), certain multiples of 1.25, certain multiples of 6, and so on. The range includes 0 and certain positive values.

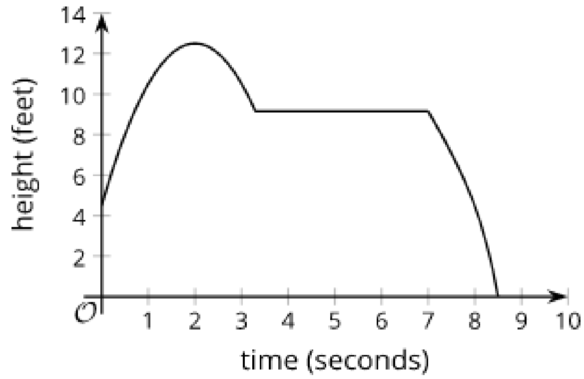
The domain of the height function would include any amount of time since the jump began, up until the jump is complete. From the graph, we can tell that this happened more than 70 seconds after the jump began, but we don't know exactly when.



The graph shows a maximum height of 80 meters and a minimum height of 10 meters. We can conclude that the range of this function includes all values that are at least 10 and at most 80.

Unit 5 Lesson 13 Practice Problems

1. A child tosses a baseball up into the air. On its way down, it gets caught in a tree for several seconds before falling back down to the ground.



   Select the **best** description of the range of this function.

   a. The range includes all numbers from 5 to 12.5.

   b. The range includes all integers between 0 and 12.5.

   c. The range includes all numbers from 0 to 8.5.

   d. The range includes all numbers from 0 to 12.5.

2. A newly planted cedar tree grows at a rate of about 3 inches per month, representing the height H after t months. The Carolina Panthers get 3 points for each field goal they make in a football game, representing their score in the game S after f field goals.

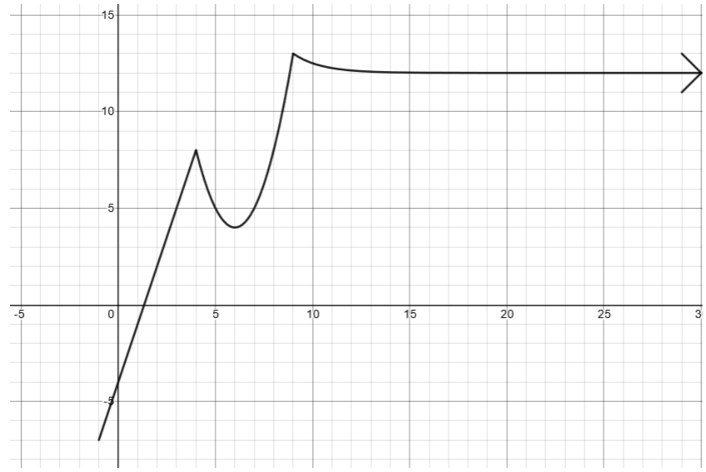
   a. What equations could represent $H(t)$ and $S(f)$?

   b. What are possible domains for $H(t)$ and $S(f)$?

   c. What are possible ranges for $H(t)$ and $S(f)$?

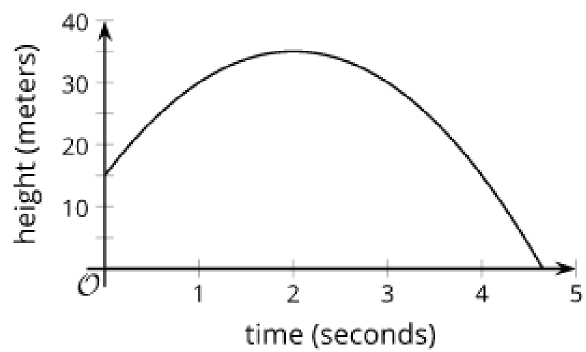
   d. Explain any differences in the domains and ranges of the two functions.

3. Based on the graph of the function, select **all** the true statements.



   a. The domain and range of the function are all real numbers.
   b. Point $(4,6)$ is a solution to the function.
   c. $y=-3$ is in the range of the function.
   d. $x=-3$ is in the domain of the function.
   e. Point $(9,13)$ is a solution to the function.
   f. The range of the function is $-7 \leq y \leq 13$.

4. The graph H shows the height, in meters, of a rocket t seconds after it was launched.



   a. Find $H(0)$. What does this value represent?

   b. Describe the domain of this function.

   c. Describe the range of this function.

   d. Find $H(x)=0$. What does this value represent?

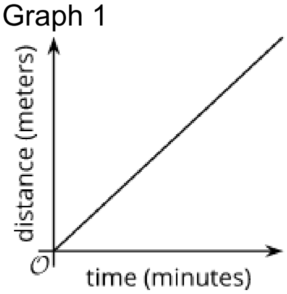
(From Unit 5, Lesson 12)

5. Lin completes a 5K using a combination of walking and running. Here are four graphs that represent four possible situations. Each graph shows the distance, in meters, as a function of time, in minutes.

   Match each description with a graph that could represent it.

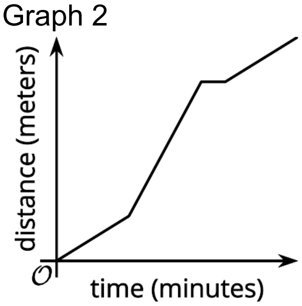
   a. Lin starts out running, but then slows down to a jog. After 10 minutes, she stops for a water break. She then runs the rest of the way.
   b. Lin starts the race walking, gradually getting faster and faster.
   c. Lin jogs at a steady pace for the entire race.
   d. Lin starts out walking, then moves to a steady run. After 15 minutes, she stops to stretch a cramped leg. Then, she walks the rest of the way.

Graph 1



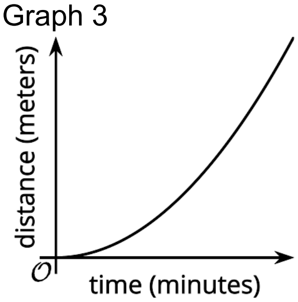
Description: _______

Graph 2



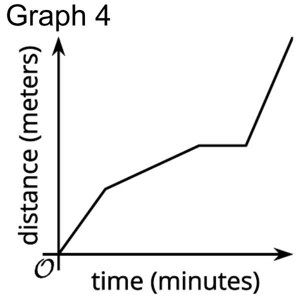
Description: _______

Graph 3



Description: _______

Graph 4



Description: _______

(From Unit 5, Lesson 7)

6. Mai has to decide between two cafeteria meal plans. Under plan A, each meal costs \$2.50. Under plan B, one month of meals costs \$30.

   a. Write an equation for function A, which gives the cost, in dollars, of buying n meals under plan A.

   b. Write an equation for function B, which gives the cost, in dollars, of buying n meals under plan B.

   c. Mai estimates that she'll buy 15 meals per month. Which meal plan should she choose? Explain your reasoning.

(From Unit 5, Lesson 5)

7. C gives the cost, in dollars, of a cafeteria meal plan as a function of the number of meals purchased, n. The function is represented by the equation $C(n) = 4 + 3n$.

   a. Find a value of n for which $C(n) = 31$.

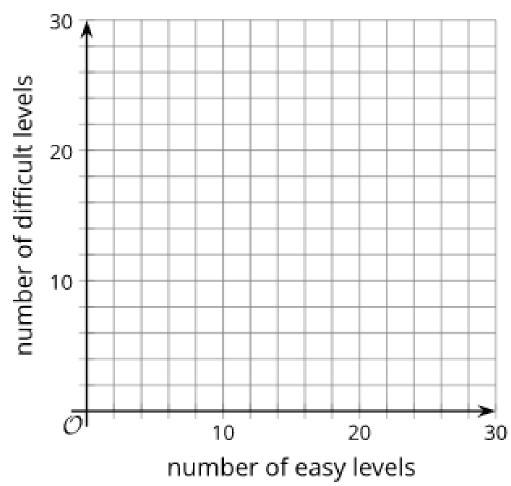
   b. What does that value of n tell you about the cafeteria meal plan?

(From Unit 5, Lesson 5)

8. Kiran is playing a video game. He earns three stars for each easy level he completes and five stars for each difficult level he completes. He completes more than 20 levels total and earns 80 or more stars.

   a. Create a system of inequalities that describes the constraints in this situation. Be sure to specify what each variable represents.

   b. Graph the inequalities and show the solution region.



   c. Then, identify a point that represents a combination of stars and levels that is a solution to the system.

   d. Interpret the point $(5, 6)$ in the context of this situation and determine how many stars Kiran earns based on this point.

(From Unit 3)

Lesson 14: Post-Test Activities

Learning Targets

- I can reflect on my progress in mathematics.
- I can share feedback that can help make me and my teacher grow.

Activity 2: Who Are They?

1. Review the "Who Are They?" pictures and predict each person's profession.

Daniel Akyeampong:	Charlotte Baidoo:	Marjorie Lee Brown:	Carlos Castillo-Chavez:
Lorin Crawford:	Tiffany Kelly:	Autumn Kent:	Susan Murphy:

2. Read over the answer key to reveal each person's profession.

3. Engage in a small-group discussion using the following prompts:
    - What made you predict the way you did?
    - Is there a specific way you think of mathematicians?
    - Who was most surprising to you and why?
    - Were any predictions correct?
    - What are the implications of making an assumption that mathematicians do not look like us?